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**The Geometrical Roots of Gothic Design and Aesthetics:
The Case of the Cologne Cathedral Choir**

The aesthetic qualities of Gothic architecture, which contribute so strongly to its social meaning and reception, have deep roots in the medieval design process. Gothic buildings typically appear restless and active, in large part because they are the physical traces of a design process involving the dynamic unfolding of geometrical figures. This essay explores that process and its visual expression, with particular attention to the case of Cologne Cathedral, one of the most spectacular and best-known products of the medieval architectural imagination (Figs. 1, 2).

The formal order characteristic of Gothic design contrasts dramatically with that of the classical design tradition established in Greco-Roman antiquity. Greek builders made little use of the arch. Instead, they typically used the post-and-lintel system, in which a series of vertical columns carries a series of horizontal beams, as seen in temples such as the Parthenon in Athens. This structural system, with its clear demarcation between supporting and supported elements, tends to convey an impression of timeless and stable order. Importantly, too, Greek builders conceptually equated their columns with human bodies, which meant that they had to observe certain basic canons of proportion. More specifically, the Greeks built in three main manners, or orders: the relatively plain columns of the Doric order seen at the Parthenon were supposed to have the robust proportions of a male warrior; Ionic columns with their scrolled capitals were to have the slimmer proportions of a woman; and Corinthian columns with their leafy capitals were slimmer still, like a young maiden. These conventions were recorded and codified by the Roman architect Vitruvius, who worked in the reign of Caesar Augustus, and whose *De Architectura* is the only treatise on building to survive from the ancient world.¹

Vitruvius acknowledged that there could be some variance of proportion within each of the orders, but he provided numerical guidelines for each type, suggesting that even the slimmest Corinthian columns should have a height-to-width ratio no greater than 10:1. To go further than this would be to break the logic of the human analogy that was central to classical architectural theory. The leafy form of the Corinthian capital suggests a different metaphor, in which the building might be compared to a growing plant, but this idea figured less centrally in Greco-Roman architecture than it later would in the Gothic era. Vitruvius himself suggested that the capital type had been invented by the Greek architect Callimachus, who had been inspired after seeing the leaves of an acanthus plant growing through the sides of a votive basket left by a young girl's grave. In this story, the leafy basket exists as a thing in itself, and in actual classical architectural practice, similarly, the leafy articulation generally remains confined to the Corinthian capitals, rather than spreading more widely across the building, as it often would in Gothic design.

During the prosperous centuries following the reign of Augustus, Roman architects developed a wide range of structural and formal possibilities far beyond those considered by

¹ On the origins and proportions of the orders, see Vitruvius, *De Architectura*, especially book IV, chapter 1. See also John Onians (1990), *Bearers of Meaning*, Princeton: Princeton University Press, pp. 8-22.

Vitruvius and his Greek predecessors. Most importantly, Roman builders began to employ arches and vaults in new and innovative ways, producing structures with vast uninterrupted interior spaces, as at the Pantheon or the baths of Diocletian in Rome, the latter now converted into the church of Santa Maria degli Angeli. Because of their curved forms, vaults created volumes that were more complex and organic than those typical of Greek architecture, but they were generally articulated with the traditional classical orders. The visual rhythms of Roman vaulted buildings also tend to be measured and slow, because the volumes are shaped by only a few large elements, instead of by many smaller ones, as would come to be the case in Gothic design. Even simpler in their spatial outlines were the boxy audience halls, with their lightweight timber roofs, that served as the main prototypes for the first large Christian churches after Emperor Constantine accepted Christianity in 313.

There is obviously no easy way to summarize the architectural history of Western Europe between the age of Constantine and the advent of the Gothic era in the twelfth century, but several basic themes deserve mention in this context. First, it would be fair to say that building practice before the millennium was largely dependent on Roman practice, although the addition of tower complexes to churches starting in the Carolingian era introduced a degree of drama to their exteriors unseen in Early Christian times. After the Millennium, stone vaulting began to be used more widely once again, but often in the context of buildings with many comparatively small spatial cells, or bays, rather than the few large ones more typically seen in great buildings from the heyday of the Roman Empire. The term “Romanesque,” coined in the nineteenth century, makes good sense in describing such architecture, in which originally Roman design elements were employed in new and unconventional ways.² Many Romanesque churches, for instance, incorporate masonry piers composed of bundles of shafts, each far slimmer than Vitruvian convention would allow. These shafts were often topped with leafy capitals of broadly Corinthian inspiration, but the proportions of the shafts themselves left the human analogy of classical column design far behind.

The early Middle Ages witnessed the emergence of a geometrically-based design tradition that would reach its fullest architectural development in the Gothic era. While the conventions of classical design govern the shape of the final object, the conventions of medieval design often involved instead the sets of procedural operations used in generating that object. They defined, metaphorically speaking, the rules of the game rather than the final score. The toolkit used by early medieval designers was small, consisting mainly of the compass and straightedge, and the geometrical operations that they undertook with them were all quite simple in themselves, but by combining these operations in sequence they could eventually produce works of dazzling complexity, including Insular manuscripts such as the Book of Kells. Recent research has demonstrated that very similar kinds of dynamic form generation were used across a wide variety of media, including the production of manuscripts, jewelry, swords, and stone sculpture.³ Architectural forms remained comparatively simple in the early Middle Ages, but by

² Eric Fernie (2014), *Romanesque Architecture*, New Haven: Yale University Press, esp. 1-8.

³ Robert Stevick (1999), “Shapes of Early Sculptured Crosses of Ireland,” *Gesta* 38, 1999, pp. 3-21. See also Robert Stevick (2004), “The Ancestry of ‘Coherent Geometry’ in Insular Designing,” *Journal of the Royal Society of Antiquaries of Ireland* 134, 2004, pp. 5-32. Peter Johnsson (2015), “The Geometry of the Medieval Sword,” in *Das Schwert: Gestalt und Gedanke/The Sword: Form and Thought*, Solingen: Deutsches Klingmuseum, pp. 16-26. Peter Johnsson (2012), “‘Righteousness is Quadrangular’ A Hypothesis on Geometric Proportions of Medieval Swords,” in *The 29th Park Lane Arms Fair Catalogue*, pp. 1-14.

the eleventh century, at the latest, similar techniques were being applied to architecture in the Anglo-Norman and French spheres.⁴

The first century of the Gothic era, between roughly 1140 and 1240, stands out as one of the most innovative periods in the history of European architecture. The building that most scholars point to as the first example of the style, Abbot Suger's Saint-Denis Abbey near Paris, already incorporates large windows, pointed arches, and a lightweight structural system supporting a ceiling made of vaults with diagonal ribs.⁵ In succeeding decades, Gothic designers would develop a largely new vocabulary of structural and decorative forms, featuring soaring spires, miniature pinnacles, and flying buttresses to help reinforce the upper walls of their churches against the thrust of their vaults. Throughout the period of its medieval flourishing, the Gothic was thus identified as the modern style; the term "Gothic" and its cognates were introduced only in the Renaissance, as misleading and dismissive slurs against this non-classical design mode.⁶ In the interim the Gothic style had achieved massive popularity. By the end of the twelfth century it had reached England, as at Canterbury Cathedral, and in the years after 1200 it became fashionable across the whole of Western Europe.

Cologne Cathedral, begun in 1248, was planned from its inception as a particularly ambitious and uncompromising sequel to the most advanced Gothic building projects seen in France up to that time.⁷ The interior of the choir, which was consecrated in 1322, rises to a height of over 43 meters (Fig. 1). The space feels tall not only because of its scale, but also because of its narrow overall proportions. The skeletal structure of the choir consists of slender shaft bundles that soar smoothly upward to the small capitals at the base of the main vault. The proportions of these shafts, of course, leave the human analogy of classicism far behind, creating an impression of transcendence. The vast windows framed by this structure include geometrical tracery: trefoils in the chapels of the first story, and more elaborate rosettes in the heads of the clerestory far above. The presence of these figures hints at the geometrical design principles that the designers of the choir would have employed while working with compasses and straightedges at their drafting tables. The rib vaults, meanwhile, seemingly slice up and subdivide the interior space, creating what art historian Paul Frankl called an aesthetics of partiality.⁸ The complexity of the interior, in which changing vistas of related but distinct parts may be glimpsed behind screens of columns and shafts, makes the cathedral seem to encapsulate the limitless richness of the cosmos itself.

An exterior view of the Cologne choir reveals more of the working structure that makes possible the skeletal appearance of its interior (Fig. 2). At ground level, slim but solid buttresses flank the windows of the chapels. Just above these window heads a horizontal cornice wraps the building like a belt, calling attention to its undulating but faceted outline. Above this level the design becomes more elaborate, as richly articulated buttresses rise upwards to support the flying buttresses bracing the walls of the upper choir. Each flyer carries a lacy row of openwork tracery

⁴ Robert Bork (forthcoming), "La géométrie de Notre-Dame de Jumièges," in James Morganstern, ed., *Notre-Dame de Jumièges*.

⁵ Robert Bork (2013), "Ground Plan Geometries in Suger's St-Denis: A Prototype for Altenberg," in Astrid Lang and Julian Jachmann, eds. *Aufmass und Diskurs*, Berlin: Lukas Verlag, pp. 55-68.

⁶ Marvin Trachtenberg (1991), "Gothic/Italian 'Gothic': Toward a Redefinition," *Journal of the Society of Architectural Historians* 50, 1991, pp. 22-37.

⁷ Arnold Wolff (1999), *The Cologne Cathedral* (Cologne: Verlag Kölner Dom); see also Christopher Wilson (1990), *The Gothic Cathedral*, London: Thames and Hudson, pp. 151-5.

⁸ Paul Frankl (1962), *Gothic Architecture*, Baltimore: Penguin, pp. 10-14.

roundels on its upper edge, echoing the interpenetration of solid and void that the flying buttress motif itself expresses. The vertical buttresses, meanwhile, become progressively slimmer as they rise, before terminating in spiky pinnacles. Small crockets carved in the shape of curling leaves seem to crawl along the upper surfaces of the buttresses, gables, and pinnacles, and budlike finials crown the pinnacle tips. These explicitly vegetal forms suggest that the cathedral itself is a living organism; this expression of the growth metaphor is far more pervasive than that introduced into some classical architecture by the use of leafy Corinthian capitals.

The history of Cologne Cathedral intertwines in complex ways with the history of its social context and scholarly reception. The project originally expressed the pretensions of Cologne's archbishops, who were among the most powerful political figures in the Holy Roman Empire, but whose relations with the local citizenry were often violent. As a result of this conflict, and because of the vast scale of the undertaking, work proceeded relatively slowly. Only the cathedral's choir was standing when the Italian humanist Petrarch visited Cologne in 1333. Petrarch championed the literary and intellectual culture of ancient Rome over that of his own day, and he thus played a crucial role in launching the movement that became the Italian Renaissance, but classical architectural conventions had not yet come into fashion in his lifetime, and he was thus able to praise Cologne Cathedral as "an uncommonly beautiful temple, which, though, still incomplete, can be called with good reason the most magnificent."⁹ In the two centuries following his visit work on the cathedral proceeded at only glacial speed, resulting in the construction of the nave aisles and the stump of a single western tower, which stood isolated from the choir when work finally ground to a stop in the sixteenth century.

As construction of Cologne Cathedral went slowly forward in the middle decades of the fifteenth century, tastemakers in Italy were turning against the Gothic architectural mode, arguing for a return to the conventions of classicism. One important contributor to this movement was Antonio di Pietro Averlino, known as Filarete, who blamed the invention of the modern style on the barbarians who sacked Rome, thus planting the seed of the idea from which the term "Gothic architecture" would grow.¹⁰ Another key figure was Leon Battista Alberti, whose expansive treatise *De Re Aedificatoria* stands as a Renaissance sequel of sorts to Vasari's *De Architectura*.¹¹ Alberti did not discuss Gothic architecture, but the emergence of the Renaissance design mode that he and Filarete championed certainly represented a challenge to the authority of the Gothic tradition.

It was in this context that the German architect Matthäus Roriczer published a short booklet describing the Gothic technique of pinnacle design; it appeared in 1486, just a year after the first printing of Alberti's treatise. Roriczer may have intended his booklet as a sort of rejoinder to the Italian work, but it was far less than comprehensive than *De Re Aedificatoria*, and while Alberti drew upon a tradition of eloquent Ciceronian rhetoric, Roriczer was drawing instead on a tradition of humble craft handbooks.¹² To provide a frame for the gritty discussion to

⁹ Petrarch, *Epistles*, Quoted in Paul Frankl (1960), *The Gothic: Sources and Literary Interpretations*, Princeton: Princeton University Press, p. 238.

¹⁰ Berthold Hub (2011), "Filarete and the East: The Renaissance of a *Prisca Architectura*," *Journal of the Society of Architectural Historians* 70, 2011, pp. 18-37.

¹¹ Leon Battista Alberti, *On the Art of Building in Ten Books*, translated by Joseph Rykwert, Neil Leach, and Robert Tavernor (1991), Cambridge, MA: MIT; see also Anthony Grafton (2002), *Leon Battista Alberti: Master Builder of the Italian Renaissance*, Cambridge, MA: Harvard University Press.

¹² Paul Crossley (1993), "The Return to the Forest: Natural Architecture and the German Past in the Age of Dürer," in T.W. Gaehtgens, ed., *Künstlerlicher Austausch. Artistic Exchange*, Berlin: Akten des XXVIII Internationalen

follow, Roriczer opens the first page of his booklet by telling his learned patron Wilhelm von Reichenau, the Bishop of Eichstätt, that his writings will “explain something of the art of geometry, beginning with first steps in extruding stonework . . . using proper measures determined with a compass.”¹³ By “extruding stonework,” Roriczer meant the extrapolation of a three-dimensional building component out of its ground plan in accord with its elevation, the process that German authors call *Auszug*, or “pulling out.”

Unlike Alberti, who wrote a text that could stand on its own, Roriczer depended on the use of images to demonstrate the meaning of his text. The left-hand side of Figure 3a, for instance, shows Roriczer’s successive steps in the generation of a pinnacle’s ground plan. This process involved the inscribing of rotated squares within the framing square, a process now frequently called quadrature. Although the process is fairly straightforward, Roriczer labeled every single point in his illustrations, describing the successive steps of the design process in numbing detail. Having established the basic square abcd in step one, for example, he explains the next step as follows: “Divide the distance from a to b into two equal parts, and mark an e at the midpoint. Do the same from b to d and mark an h; from d to c and mark an f; from c to a and mark a g. Then draw lines from e to h, h to f, f to g, and g to e, as in the example of the figure drawn hereafter. . . .”¹⁴ Despite its tediously explicit specificity, Roriczer’s text is all but unintelligible without reference to his illustrations. In consulting those graphics, though, the nature of Roriczer’s process becomes clear: after defining the details of the pinnacle’s ground plan with a series of operations easily performable with the compass and rule, as seen on the left of Figure 3a, he defines its elevation by stacking a series of modules based on the size of the plan.

As these graphics show, then, the three-dimensional shape of the pinnacle results from the extrusion of the plan according to the elevation. This general idea also applies to the shape of larger Gothic building elements, and even entire churches, as subsequent discussion of Cologne Cathedral will demonstrate. In an important sense, therefore, Gothic design might be described as two-plus-two dimensional. In ancient and Renaissance design, by contrast, basic three-dimensional volumes such as spheres and cubes often had conceptual primacy. Because Gothic design largely lacked reference to such basic and familiar forms, its geometrical logic has often seemed inscrutable to scholars and commentators from the Renaissance onwards.

Another obstacle to the understanding of Gothic design has been its relative lack of standardization. The pinnacle that Roriczer illustrated was just one of a potentially infinite series of such pinnacles, each one of which would be the result of slightly different decisions taken in the dynamically unfolding design process. Gothic design elements, in other words, tended to be customized. There was thus great scope for originality in the Gothic design tradition. Conversely, though, Gothic design made less overt appeal to notions of correctness and convention. For this reason, in part, Gothic design came to be seen as less authoritative than classical design, whose conventions began to be widely known and understood in the Renaissance.

An important factor in driving the Gothic-to-Renaissance transition in architecture was the publication of illustrated treatises on classical design. Among the most influential of these

Kongresses für Kunstgeschichte, pp. 71-80.

¹³ Lon Shelby (1977), *Gothic Design Techniques: the fifteenth-century design booklets of Mathes Roriczer and Hanns Schmuttermayer*, Carbondale: Southern Illinois University, p. 83.

¹⁴ Quoted from Shelby, *Gothic Design Techniques*, p. 85.

were the *Libri* of Sebastiano Serlio, whose publication began in 1537.¹⁵ Figure 3b shows Serlio's illustration of typical columns belonging to the five major orders recognized in the Renaissance: the Tuscan order, a cousin of the Doric, appears at left, followed by the Doric, Ionic, Corinthian, and finally the composite, a variant of the Corinthian that incorporates quasi-Ionic scrolls in its capitals. Each illustrated column and its corresponding base is labeled with a number describing the canonical ratio of its height to its width. As simple graphics such as these made the conventionally correct column forms accessible to a broad European audience, the more complicated procedural order of Gothic design began to fall out of fashion.

Perhaps the single most influential condemnation of Gothic design practice came from Giorgio Vasari, the sixteenth-century Florentine artist and author widely regarded as the father of art-historical writing. In the preface of his *Vite*, or *Lives of the Artists*, first published in 1550, Vasari offered a scathing critique of the Gothic design mode, whose eclipse he cheered. From his perspective, the Gothic mode no longer appeared modern; instead, he associated it with the Germans, as the following famous passage indicates:

There are works of another sort that are called German, which differ greatly in ornament and proportion from the antique and the modern. Today they are not employed by distinguished architects but are avoided by them as monstrous and barbarous, since they ignore every familiar idea of order; which one can rather call confusion and disorder, for in their buildings, of which there are so many that they have contaminated the whole world, they made portals adorned with thin columns twisted in corkscrew fashion (vine tendrils), which do not have the strength to support a burden, however light. And so, above all their facades and their other decorative parts, they built one cursed tabernacle on top of the other, with so many pyramids and points and leaves that they do not stand, as it appears, not to mention being able to hold themselves up, and they have more the quantity of seeming to have been made of paper, than of stone or marble. And in these works, they made so many projections, openings, little consoles, and twining vines, that they threw the works that they built out of proportion; and often they reached such a height, by placing one thing on top of another, that the end of a door touched its roof. This manner was invented by the Goths, who, after the destruction of the ancient buildings and the dying out of architects because of the wars, afterwards built—those who survived—edifices in this manner.¹⁶

In conflating recent northern European building practices with the ancient Goths who had sacked Rome, Vasari was building on ideas introduced by Filarete, creating a suitably negative origin myth for the mode that Italian Renaissance tastemakers had rejected, and obscuring from historical memory the fact that the Gothic mode had actually arisen in France, something that he likely did not know himself.¹⁷ In suggesting that Gothic architecture was disorderly, Vasari was

¹⁵ Sebastiano Serlio, *I Sette libri dell'architettura*; see *Sebastiano Serlio on Architecture Volume One: Books I-V of 'Tutte L'Opere D'Architettura et Prospetiva'*, ed. by Vaughan Hart and Peter Hicks (1996), New Haven: Yale; and *Sebastiano Serlio on Architecture Volume Two: Books VI and VII of 'Tutte L'Opere D'Architettura et Prospetiva'*, with 'Castrametation of the Romans' and 'The Extraordinary Book of Doors,' ed. by Vaughan Hart and Peter Hicks, (2001), New Haven: Yale.

¹⁶ Vasari, *Vite*, quoted in Frankl (1960), pp. 290–291.

¹⁷ Anne-Marie Sankovitch (2001), "The Myth of the 'Myth of the Medieval': Gothic Architecture in Vasari's Rinascita and Panofsky's Renaissance." *RES* 40, 2001, pp. 28-50.

similarly misleading. It was true, of course, that Gothic designers had not followed the conventions of classicism, but that hardly meant that their buildings were undisciplined.

Gothic design received little sympathetic attention in the centuries following the Renaissance, since classical design had become closely associated with both good taste and political power. Indeed, the largest single factor in spreading the Renaissance mode throughout Europe was surely its association with the glory of the Roman Empire, which many subsequent rulers sought to emulate. It was only in the eighteenth century that authors and critics began to develop interpretive frameworks within which the Gothic could be more readily appreciated. In the years around 1700, for example, the Englishmen John Ashley-Cooper and John Dennis began to write about the power of vast and inhuman vistas like those seen in the Alps, which evoked awe and even fear rather than simple pleasure. Both Edmund Burke and Immanuel Kant would further develop this idea of the Sublime, which became an important element of aesthetic theory. Meanwhile, Horace Walpole began in the 1750s to incorporate Gothic elements in the remodeling of his residence, Strawberry Hill, and two decades later the young Goethe wrote *Von Deutscher Baukunst*, a short essay on Strasbourg Cathedral that celebrated its presumed architect, Erwin von Steinbach, as an exemplar of German national genius. By the end of the eighteenth century, therefore, currents of Gothic revivalism had begun to emerge in both England and the German world, but careful analysis of Gothic design practice was still lacking.¹⁸

Cologne Cathedral began to figure strongly in the discussion of Gothic design and aesthetics especially in the decades around 1800. A key figure in this conversation was Georg Forster, who wrote in 1790 that he visited the cathedral “in order to feel thrill of the sublime...Extended to an enormous length the groups of slender piers stand like the trees of a primeval forest... Though the immeasurability of the universe cannot be illustrated in a finite space, there is nevertheless in this bold upward soaring of piers and walls a ceaseless energy that the imagination so easily prolongs into the illimitable.”¹⁹ Building on Forster’s comments, Friedrich von Schlegel in 1805 wrote that “if the whole, from the outside, with all its countless towers and turrets, looks at a distance not unlike a forest, so the entire growth, when one approaches somewhat more closely, seems rather to be comparable to an immeasurable formation of a crystallized nature.”²⁰ These two intuitively derived metaphors, of organic and crystalline growth, provide surprisingly good models for the kind of dynamic geometrical practice used by the Cologne designers, as subsequent discussion will show.

Cologne Cathedral gained particular renown after the end of the Napoleonic Wars, thanks largely to the efforts of Sulpiz Boisserée, who in 1821 began to publish an impressive series of volumes urging the completion of the cathedral, and including illustrations based on careful measurements of its fabric. Figure 4a, for example, shows a cross-section of the choir prepared by the engraver Christian Friedrich Traugott Duttonhofer.²¹ Further illustrations showed how the cathedral would appear with its western façade completed with twin spires, based on a

¹⁸ On these developments, see, for example, Georg Germann (1972), *Gothic Revival in Europe and Britain: Sources, Influences and Ideas*, Cambridge, MA: MIT.

¹⁹ Forster, quoted in Frankl (1960), p. 444.

²⁰ Schlegel, quoted in Frankl (1960), pp. 462-3.

²¹ Figure 4a reproduces Tafel VI from Sulpiz Boisserée (1821-1831), . See also (1823), *Geschichte und Beschreibung des Doms von Köln*, Stuttgart: Cotta.

spectacular medieval design drawing that had been rediscovered in 1814.²² Boisserée's advocacy helped to make the completion of the cathedral into a popular goal, which was realized successfully in a building campaign lasting from 1842 to 1880, with strong support from both local donors and the national government of Prussia, which controlled the Rhineland from 1815 onwards.

The nineteenth-century builders of Cologne Cathedral worked at a very high level of archaeological precision, as did many of their colleagues in the Gothic Revival movement, but scholarly understanding of genuine medieval design techniques has remained surprisingly imprecise until relatively recent years. Roriczer's handbook of pinnacle design was republished in the 1840s, but its relevance for the larger questions of building design seemed limited. A whole range of scholars working in the late nineteenth and early twentieth centuries attempted to develop geometrical explanations for Gothic building designs, but consensus on these questions remained elusive. In a series of influential articles from the years around 1970, Konrad Hecht even argued that Gothic architects used simple modules and numerical ratios, rather than dynamic geometry, in the development of their building plans.²³ It has subsequently become clear that Gothic builders actually used both of these strategies; in fact, the numerical ratios Hecht cited were often arithmetical approximations to quantities originally conceived geometrically. The ratios 7:5 and 12:7, for example, provide good approximations to the geometrically derived $\sqrt{2}$ ratio between the diagonal of a square and its side. Recent scholarship has clearly demonstrated the conceptual primacy of geometry in the Gothic design process.²⁴

Careful analysis of the Cologne Cathedral choir shows that Gothic builders approached the design of whole churches in the same basic way that Roriczer approached the design of pinnacles: by defining the ground plan first, and then projecting its elements into the third dimension, while using dynamic geometry to set the proportions of figures in both the horizontal and vertical planes. To set the stage for consideration of this case, Figure 4 indicates the relationship between a recently produced ground plan of the Cologne choir, below, and the corresponding elevation as engraved by Duttonhofer, above. The ground plan shows that the east end of the church sits on a perfectly semicircular plinth whose radius at ground level is 24.15m.²⁵ This radius equals the diagonal of a square measuring 17.08m on a side, as the right half of Figure 4b shows. It will be convenient to call the latter dimension 1 unit, as on the left side of the figure. On this plinth, seven small chapels gather around the central apse and its ambulatory, creating a fan-shaped array whose faceted outline was already seen from the exterior in Figure 2. Five of the chapels lie wholly to the east of the array's center point, above the baseline of the

²² See Marc Steinmann (2003), *Die Westfassade des Kölner Domes: Der mittelalterliche Fassadenplan F*, Cologne: Verlag Kölner Dom. For the geometry of the drawing, see Robert Bork (2011), *The Geometry of Creation*, Farnham: Ashgate, pp. 113-21.

²³ Hecht's *Maß und Zahl in der gotischen Baukunst* first appeared as three successive issues of *Abhandlungen der Braunschweigischen Wissenschaftlichen Gesellschaft*: 21 (1969), 22 (1970), and 23 (1970). The complete study has been republished as a single volume by Georg Olms Verlag (Hildesheim, 1979).

²⁴ Bork (2011). On the use of modules as approximations to geometrically derived quantities, see also Franklin Toker (1985), "Gothic Architecture by Remote Control: An Illustrated Building Contract of 1340," *Art Bulletin* 67, 1985, pp. 67-95; and Norbert Nussbaum and Sabine Lepsky (2012), *Gotische Konstruktion und Baupraxis an der Zisterzienserkirche Altenberg, Band 2 Quer- und Langhaus*, Bergisch Gladbach: Altenberg Dom-Verein.

²⁵ All the dimensions cited in this essay are locked to each other through precise geometrical relationships modeled using the Vectorworks CAD system. Given the 24.15m radius of the plinth, therefore, all of the other dimensions shown can be derived by the steps described in the text.

graphic, while two others are centered on that baseline. The number of chapels in the semicircle is thus five and two halves, or six. Each chapel thus occupies a wedge of space occupying one sixth of a semicircle, or one twelfth of a full circle.

Figure 5 illustrates two sets of geometrical constructions relevant for the design of the Cologne choir. The left-hand column shows figures related to the square, while the right-hand column shows the corresponding figures related to the twelve-sided dodecagon. The square figures correspond to those illustrated in Roriczer's pinnacle plans. The upper left graphic shows that, when one inscribes a circle within a square, and then a smaller square within that circle, the side length of the small square will be .707 as great as that of the original framing square, where $.707 = 1/\sqrt{2} = 1/(1.414)$. In modern mathematical parlance, one would say that .707 is the cosine of the 45-degree angle between the equator of the original square and the line to its corner, but medieval builders would not have used any such notation; instead, they would simply have used their compasses and straightedges to draw these simple interlocking figures. It will be convenient to call this scaling by .707 a quadrature factor. In the bottom left figure, the same procedure has been repeated a second time, as the two sets of shaded arcs show, producing a new and still smaller square whose side measures $.500 = (.707)^2$ as long as the original framing square. Precisely analogous operations can be performed using the geometry of the dodecagon. As the upper right graphic shows, a dodecagon inscribed within a circle within a framing dodecagon will be smaller than the framing figure by a dodecature factor of .966, which is the cosine of the 15-degree wedge between its equator and its corner. Finally, as the bottom right graphic indicates, a second iteration of this operation will produce a new figure whose diameter between the shaded arcs will be $.933 = (.966)^2$ as great as the diameter of the original figure. These relationships play fundamental roles in establishing the proportions of the Cologne choir, both in plan and in elevation.

As the bottom of Figure 6 shows, many of the key dimensions in the Cologne ground arise from application of these procedures to the basic plinth forms already seen in Figure 4b. The radius of the circle swinging through the centers of the engaged piers between the chapels, for example, is 15.94m, or .933 units, which is to say that it is smaller than the plinth radius by one factor quadrature factor and two dodecature factors. Stepping in by a third dodecature factor one finds squares 15.39m or .901 units on a side; their outer corners locate the outboard surfaces of the large buttresses between the chapels. A fourth such step inwards gives a radius of 14.87m, or .870 units. The pier centers of the main choir vessel lie half this distance, or .435 units, from the building centerline. The width between the main pier axes of the choir is thus 14.87m, which can be defined as one italicized *unit*, as the label just above the choir plan indicates.

With the crucial main vessel width established in the ground plan, the overall logic of the choir elevation quickly begins to come into focus, as the upper portion of Figure 6 shows. A great octagon with facets of this length rises to a height of 2.414 *units*, or 35.89m. A rotated square framing this octagon has its upper tip at a height of 2.914 *units*, or 43.32m, locating the bottom surface of the main vault keystones. The overall height-to-width ratio of the cathedral's main is thus set by this simple crystalline figure. The lateral corners of the rotated square, moreover, align perfectly with the vertical axes of the outermost pinnacles on the buttress superstructure. These are particularly interesting results given the dramatic changes of articulation style previously noted between the lower and upper sections of the choir. Clearly the builders of the upper choir were being careful to respect and visually express a geometrical scheme derived from the dimensions of the ground plan, rather than just stretching the elevation

upwards to arbitrary heights. Further evidence for this claim comes from the height of the roofs over the side aisles, which at 25.38m or 1.707 *units* matches the height of the great octagon's upper lateral corners.

The more closely one looks at the plan and elevation of the Cologne choir, in fact, the more one appreciates the precision of the geometrical dialog between them. In the lower portion of Figure 7, lines have been drawn through the points where the rays through the chapel buttresses intersect the semicircular outline of the plinth. When these new lines are extended to meet the baseline of the composition, they establish its full width between the outer buttress faces, which is 53.87m. In the upper portion of Figure 7, this dimension has been taken as the diameter of a large half-dodecagon centered on the midpoint of the choir floor. The top facet of this new figure lies at a height of 26.94m, or 1.812 *units*, aligning precisely with a prominent horizontal molding on the outboard buttresses. Even more importantly, the location of the figure's next lower corner locates the top edge of the first interior story, at a height of 19.72m, or 1.326 *units*. The same height corresponds to the break between the lower and simpler part of the outer buttresses, and the more complex buttress superstructure.

Figure 8 adds some final valuable details to this picture. In the ground plan, vertical lines have been drawn through the previously mentioned points where the rays between the chapels intersect the semicircle of the plinth. These verticals, which are 46.66m apart, establish the distance between the outer wall surfaces of the choir aisles. Diagonal lines framed by these verticals pass precisely along the lines describing the stained glass windows in the first, fourth, and seventh chapels. As further lines in the fourth chapel show, the width of the chapel is set by the already-established location of the axial piers framing its entrance, while the chapel itself has the outline of a regular octagon framed by these lines. Analogous constructions, of course, could define all of the other chapels and their facets.

In the upper part of Figure 8 two new systems have been added, one governing the interior, the other governing the exterior. On the interior, two equilateral triangles have been added between the axes of the main piers. The first, placed on the groundline, reaches a height of 12.87m, or .866 *units*, establishing the location of the main arcade capitals. The second, placed on the already established story division at height 19.72m, reaches to a height of 32.59m, or 2.192 *units*, establishing the location of the high capitals from which the main vaults spring.²⁶ The shape of the exterior superstructure, meanwhile, is set by a huge new dodecagon, whose lower facet, measuring one *unit*, corresponds to the floor of the main choir vessel. The top facet of this figure lies at a height of 55.48m, or 3.731 *units*, locating the tips of the choir's uppermost pinnacles, which are the highest masonry elements in the main body of the church. The tips of the outer two pairs of pinnacles, which lie just above and just below the 43m height of the main vault, fit neatly within the circle framing this great dodecagon. Tangents to this circle parallel to the facets of the figure converge at height 60.90m, or 4.095 *units*, locating the top of the timber roof truss.

As the preceding discussion has shown, an elegant and precise geometrical system establishes the proportions of the Cologne Cathedral choir in both plan and elevation. Vasari was surely wrong, therefore, to dismiss such buildings confused and disorderly. Instead of having the straightforward and largely static order seen in ancient Greco-Roman buildings and their Renaissance sequels, however, Gothic buildings stand as the physical traces of dynamically

²⁶ The top edge of the triforium falls halfway up this second triangle.

unfolding geometrical growth processes. Romantic authors such as Forster and Schlegel thus had good reason to compare the Cologne choir to growing systems in nature, including trees and mineral crystals. Because the Gothic design process can be so complicated, and because its products are so diverse, detailed understanding of its principles has emerged only slowly. In recent years, however, analyses such as the one presented here have clarified both the methods used by Gothic designers, and their relationship to other medieval craft traditions. Now that this picture has come into sharper focus, such evidence will begin hopefully to enrich the broader scholarly conversation about Gothic and the arts.

Figure 1



Figure 2



Figure 3

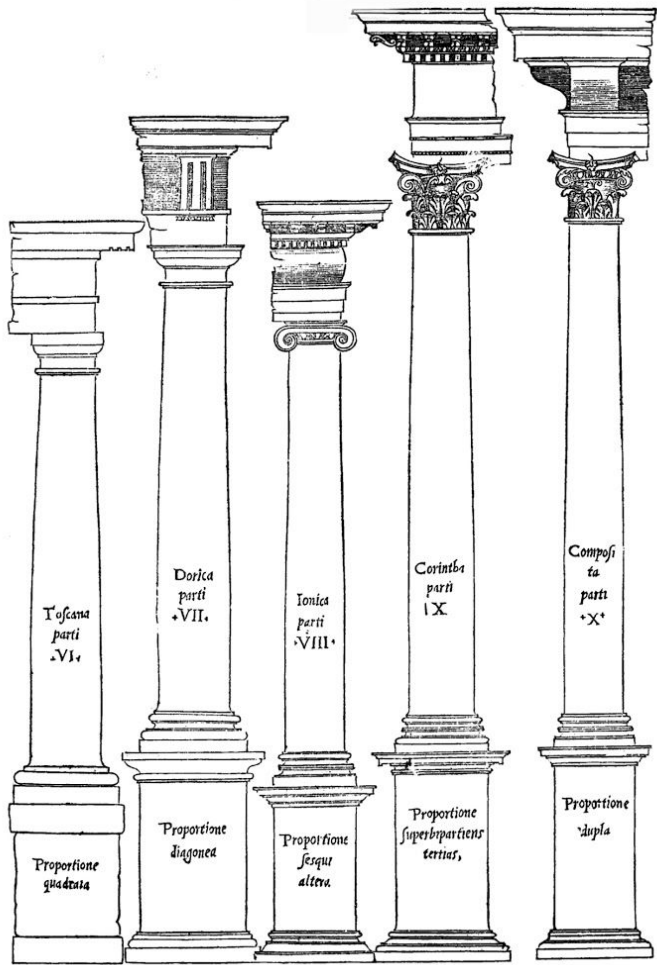
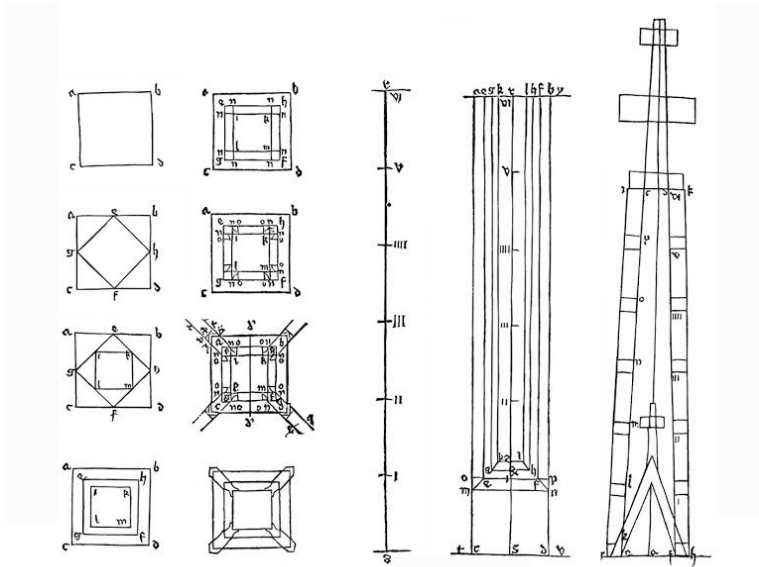


Figure 4

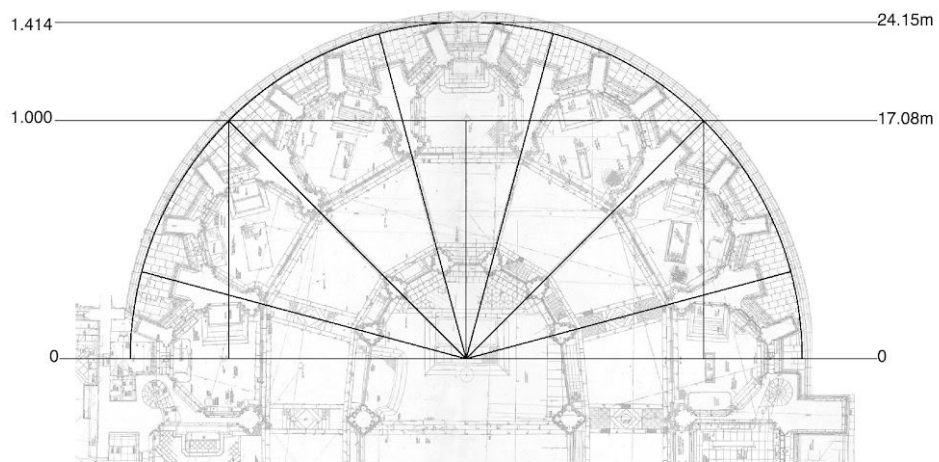
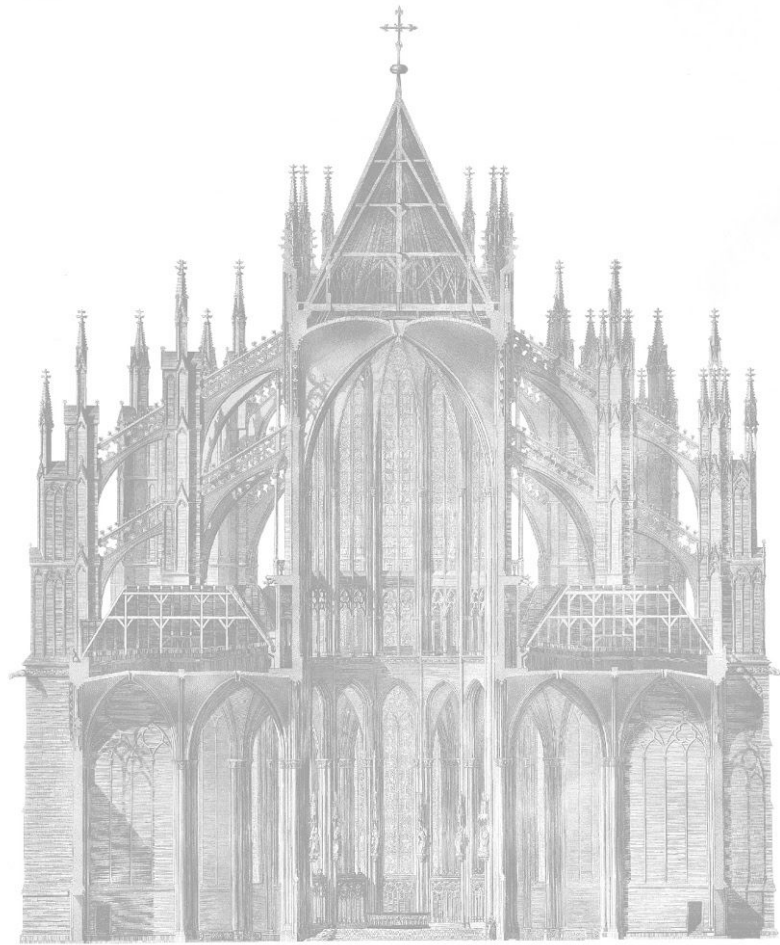


Figure 5

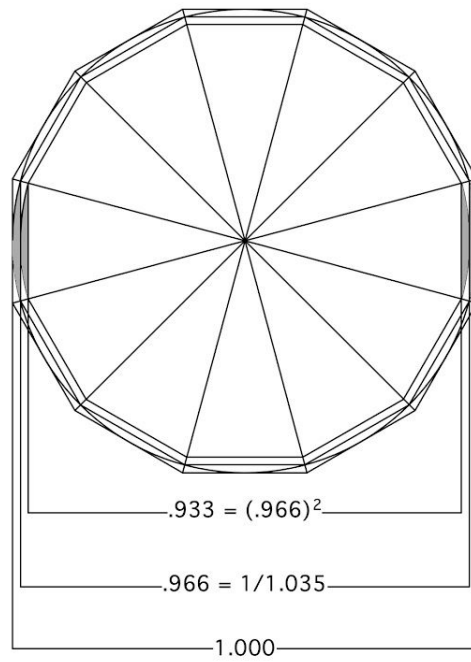
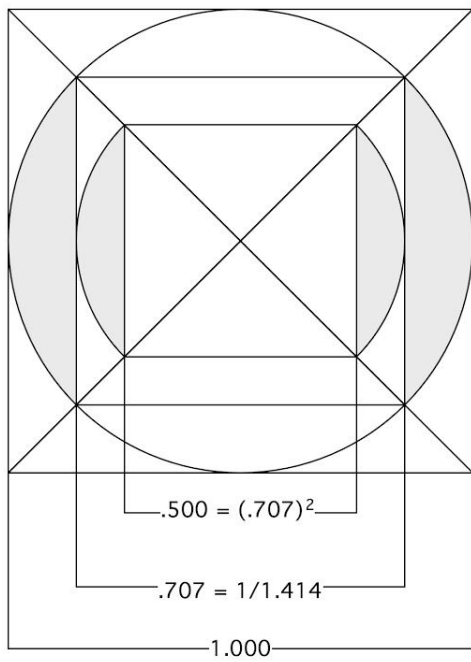
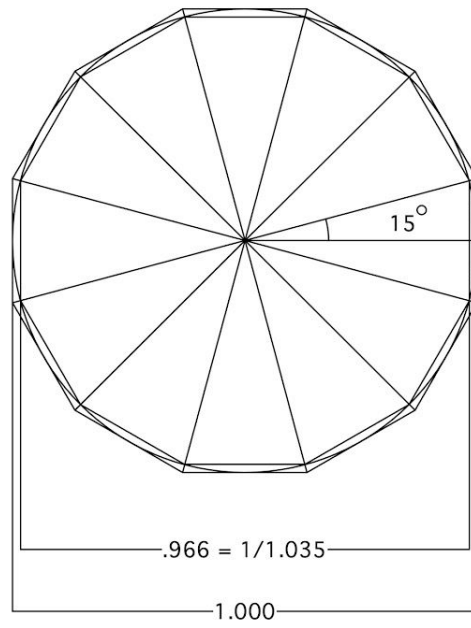
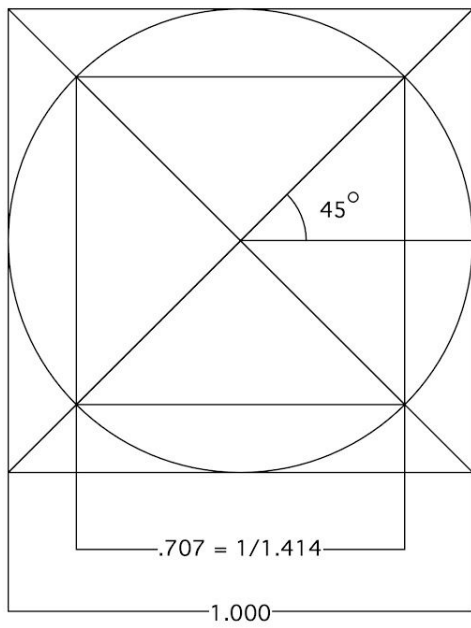


Figure 6

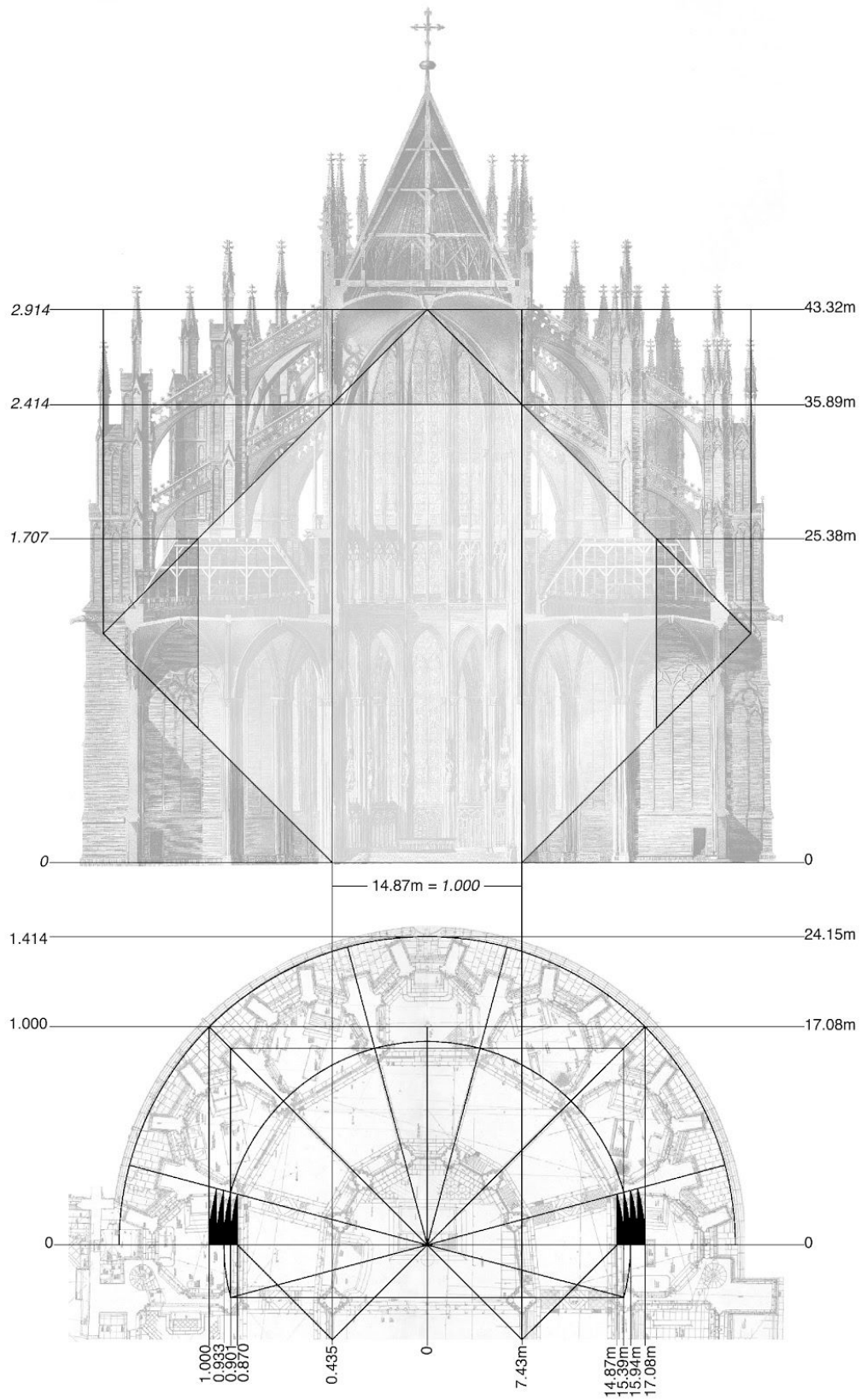


Figure 7

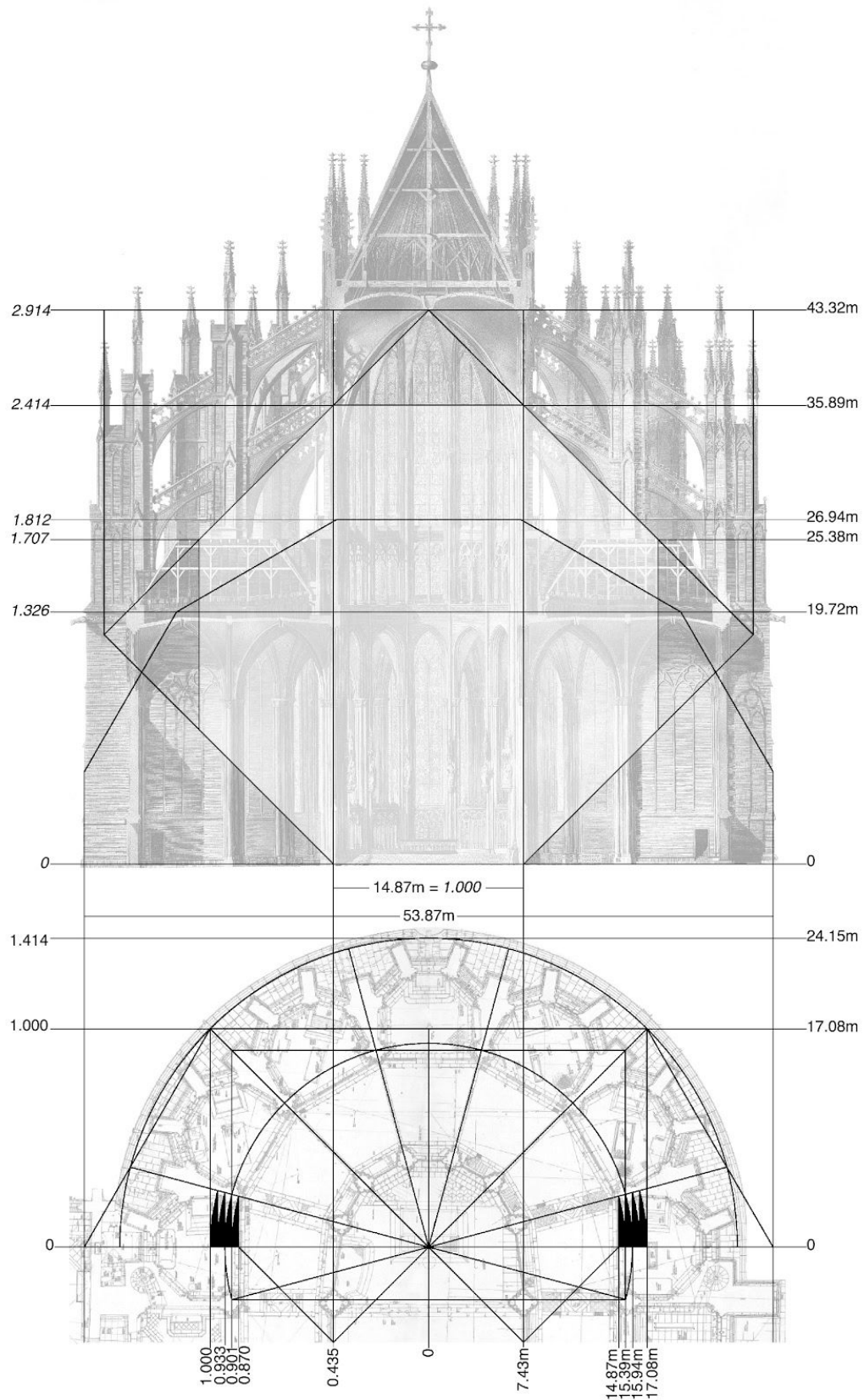


Figure 8

