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A Geometrical Analysis of the Gothic Choir at Aachen Cathedral

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The design of the Gothic choir at Aachen Cathedral can be readily understood in geometrical terms. Because the elevation generally develops out of the ground plan, it makes sense to begin with the latter. Like all complex Gothic structures, the choir can be understood as the physical trace of a dynamic design process, which generates intricate forms through combination of only a few geometrical procedures. Among the most important of the procedures in this widespread and long-lasting lingua franca of Gothic designers were the nesting and rotation of polygons, especially triangles, squares, and octagons, and the use of the Golden Section.[1] In the case of the Aachen choir, the design seems to have been conceived starting in the crucial area where the radial geometry of the hemicycle meets the axial geometry of the straight bays.

Ground Plan (see Figure 1)

The ground plan involves almost exclusively operations based on squares and circles. Out of these simple ingredients an elegant scheme soon develops.

Red lines:
We begin the development of the design in the center of the choir polygon, at the position of the keystone. The transversal rib that separates the polygon from the straight bays lies 3.155m west of this origin point. Four squares with this side length can be grouped around the north-south axis through this origin, so as to form a 2x2 grid. In this way, they together form a larger square 6.310m on a side. The half-diagonal of this larger square, which runs at an angle of 26.56°, measures 7.055m.

Orange lines:
This 7.055m-long half-diagonal defines the radius of a circle that sweeps through the interior corners of the hemicycle wall. Each of the seven eastern facets of the hemicycle has an angular width of 180°/7 = 25.71°. The span of the straight choir bays, measured between the window surfaces, equals the diameter of the hemicycle, which is thus 2 x 7.055m = 14.110m.

Golden rectangle:

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[1] See, for example: Nancy Wu (ed.), *Ad quadratum. The Practical Application of Geometry in Medieval Architecture* (Aldershot, 2002); and Robert Bork, *The Geometry of Creation: Architectural Drawing and the Dynamics of Gothic Design* (Farnham, 2011).

The parts of the plan described so far together define a rectangle with the proportions of the Golden Section. This rectangle measures 7.055m to the east of the origin line, and 3.155m to its west, for a total length of 10.210m, while its width of 6.310m matches that of the red double square. The proportions of the square are thus 10.210/6.310 = 1.618. This perfect Golden Section proportion results from the unfolding of the diagonals in the red double square.

Yellow lines:
The westernmost ribs of the hemicycle connect its keystone to points 3.155m to its west and 7.055m off the choir's longitudinal axis. These ribs thus run at angles of 24.09° from the choir's cross-axis, and they have a length of 7.728m. A circle of this radius circumscribes the 14-sided polygon whose seven eastern facets define the location of the seven eastern windows in the hemicycle.

Green lines:
The two straight choir bays can be understood in principle as pairs of squares each measuring 7.055m. The free span of the main choir vessel between its window surfaces, as noted previously, is therefore 2 x 7.055m = 14.110m. The angle of the diagonal ribs in the straight bays measures 26.56°, in accord with the proportions of the double square, except in the southwestern corner, where the ideal geometry is compromised by the intrusion of a stair turret.

Blue lines:
When a blue circle is circumsribed around the great green square framing the straight bays, one finds that its radius is 14.110 x $\sqrt{2}$ = 19.954m. This blue circle defines the width of the main buttresses in the straight bays, measured across their outer surfaces, although this relationship is slightly compromised by imprecise construction on the north side. A second circle of equal size centered on the hemicycle keystone defines the depth of the buttresses in the polygonal apse.

In sum, therefore, a few simple geometrical operations suffice to define the ground plan of the Aachen choir.

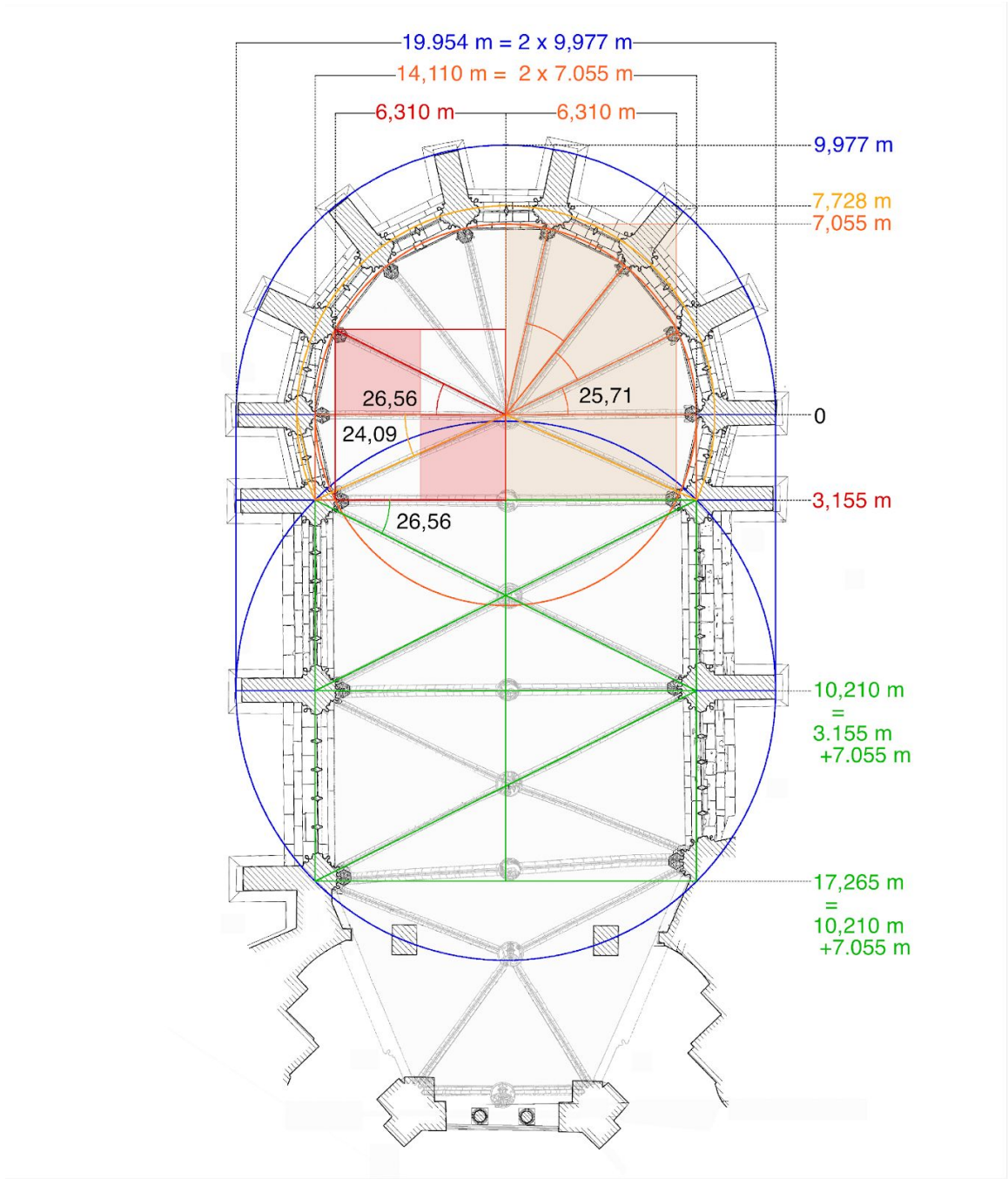

Figure 1
Aachen Cathedral, ground plan of Gothic choir with geometrical overlay.

Elevation (see Figure 2)

While the ground plan of the choir evolves principally out of geometries based on the square and the circle, the elevation evolves instead from combinations of circles and octagons, a combination often seen in Gothic elevation design.

The analysis of the elevation presented here takes into account the archaeological discovery that the floor level of the choir originally lay approximately 80cm lower than it does today.[2] The geometrical constructions proposed here thus take that original level as their baseline.

Red lines

The overall framework of the elevation is set by a great red octagon, whose facet length of 14.110m corresponds to the width of the choir straight bays as measured between their window surfaces. This figure reaches a height of 34.063m, establishing the height of the wall up to the base of the small circulation gallerie on the exterior. A circle inscribed within this octagon intersects the lines connecting its corners at a height of 32.768m. This level corresponds to the top of the vault surfaces, and to the location of the gargoyles that protrude from the buttresses. The shared equator of the circle and octagon, at height 17.031m, locates the top of the heavy round reinforcing arch in the western bays that carry the diagonally planted walls above. The midpoints of the lower diagonal octagon facets, at height 4.988m establish the bottom margin of the drip molding as the base of the exterior window sills.

Although the Aachen choir was certainly modeled after the Sainte-Chapelle in Paris, its proportions diverged markedly from those of its prototype. As Stephen Murray has shown, the Sainte-Chapelle has a width-to-height ratio of 1:2, like a stack of two squares.[3] These proportions can also be seen in other buildings closely associated with Parisian architecture of the mid-13th century, including the naves of St-Denis Abbey and Strasbourg Cathedral. If such proportions had been established at Aachen, based on the width of the choir vessel, the height of the vaults would have been 28.220m. The considerably taller proportions of the actual structure correspond instead to a series of octagon-based elevation and façade designs traceable from 13th-century Reims to the start of the 16th century.[4]

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[2] Joseph Buchkremer, "Die Sicherungsarbeiten an der gotischen Chorhalle der Münsterkirche in Aachen," *Denkmalpflege und Heimatschutz* 30 (1928), pp. 1-13, esp. 4.

[3] Stephen Murray, "The Architectural Envelope of the Sainte-Chapelle," *AVISTA Forum* 10.1 (1996/1997), pp. 21-25.

[4] Great octagons help to govern the elevations in Reims Cathedral, in the so-called Reims Palimpsest, in the Liebfrauenkirche in Trier, in the choir of Prague Cathedral, in the choir and façade of the Cistercian church at Altenberg, in the façade of Orvieto Cathedral, and in an unexecuted façade design for the cathedral of Clermont Ferrand. Stacked octagons also figure in the tower designs for Cologne Cathedral and the great parish churches of Freiburg-im-Breisgau and Ulm. See Bork, *The Geometry of Creation*, and Sabine Lepsky and Norbert Nußbaum, *Gotische Konstruktion und Baupraxis an der Zisterzienserkirche Altenberg, Bd. 2: Quer- und Langhaus* (Bergisch-Gladbach, 2012), pp. 80-88.

Orange lines:
The total width of the Aachen choir structure, which measures 19.994m to the outer surfaces of its buttresses, can be taken as the side length of a large orange square. The center of this square, at height 9.977m, matches the height of the arch springers in the radial walls of the Carolingian octagon; it is not clear whether this relationship was coincidental, or whether it was deliberately contrived to help link the new choir to the older structure. It is clearly significant, however, the top edge of the square establishes the baseline of the window story in the western facets of the choir. Slightly further up, at height 21.164m, the verticals flanking the orange square cut the red radii of the great octagon. This level, which is marked by a blue horizontal, establishes not only the springline of the choir vaults on the interior, but also the height of a setback on the exterior buttresses, and the location of the uppermost iron ring anchor that helps stabilize the structure and its expansive glass walls. The sides of the orange square intersect the circle inscribed within the large red octagon at height 3.228m, the level of the drip molding on the lowest setback on the buttresses. The sides of the square also intersect the diagonals that pass through the center of the great octagon at height 7.066, establishing the baseline of the spired canopies that frame the apostle figures on the interior walls of the choir.

Yellow lines:
Within the orange square just described, one can inscribe a yellow octagon, whose facets will measure 8.265m. A yellow circle inscribed within this octagon will cut its corner-to-corner radii at a height of 13.795m. This level, whether coincidentally or deliberately, coincides closely with the top edge of the horizontal bridge in the columnar screens of the Carolingian building. This level also marks the baseline of the small cusped arches that articulat the lower tracery transoms in the Gothic choir windows. Further below, the yellow circle cuts two other rays of the yellow octagon at height 6.159m, the level that serves as the baseline of the choir windows, following two successive Gothic redesigns of the sockel zone.[5] Because these levels both correlate well with the window structure, and because both arise from the same basic geometrical scheme, it seems likely that the current tracery transoms retain the location of the originals, which were dismantled in 1779.[6]

Green lines:
Green diagonals rising from the corners of the red framing octagon cut the corner-to-corner radii of the yellow octagon at height 9.121m, establishing the level of the second horizontal molding on the external buttress faces. The next molding above lies at height 14.965m, where the diagonals of the orange square intersect the corner-to-corner radii of the red octagon. This level, equivalently, can be found by striking a green diagonal up from the equator of the orange square until it meets the diagonals of the square, which shows that this height is simply ¾ of the square's 19.954m height.

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[5] See Ulrike Heckner, "Die gotische Chorhalle des Aachener Doms: Ergebnisse der Bauforschung," in *Die gotische Chorhalle des Aachener Domes und ihre Ausstattung. Baugeschichte – Bauforschung – Sanierung*, (Petersberg, 2002), pp. 115-125.
[6] The tracery of the choir was then rebuilt in the middle of the 19th century, but not precisely following the original Gothic format. See Ingeborg Schild, *Die Baumeister am Aachener Dom im 19. Jahrhundert* (Aachen, 1983), pp. 10-15.

To set the stage for the construction of the blind tracery on the west facet of the choir, it is convenient to establish a green octagon whose total width matches the 8.265m facet width of the yellow octagon, and whose center lies on the upper side of the orange octagon. A green rotated square can then be circumscribed within that octagon.

Blue lines:
An octagon inscribed within the green rotated square has a diameter of 5.844m. This is the width of the blind window in the western facet of the choir, measured between the axes of its tracery mullions. The lateral facets of the blue octagon end at height 21.164m, the same level as the springing of the choir vaults, which was previously found using the orange square and the radii of the red octagon.

Virtually all of the main elements of the elevation are similarly overdetermined. This fact attests to the coherence of the formal system that grows out of the simple ground plan geometry, a coherence that extends from the overall outlines of the elevation to its details. This can be seen with particular clarity in the design of the blind western window, which includes the only original Gothic tracery in the choir.

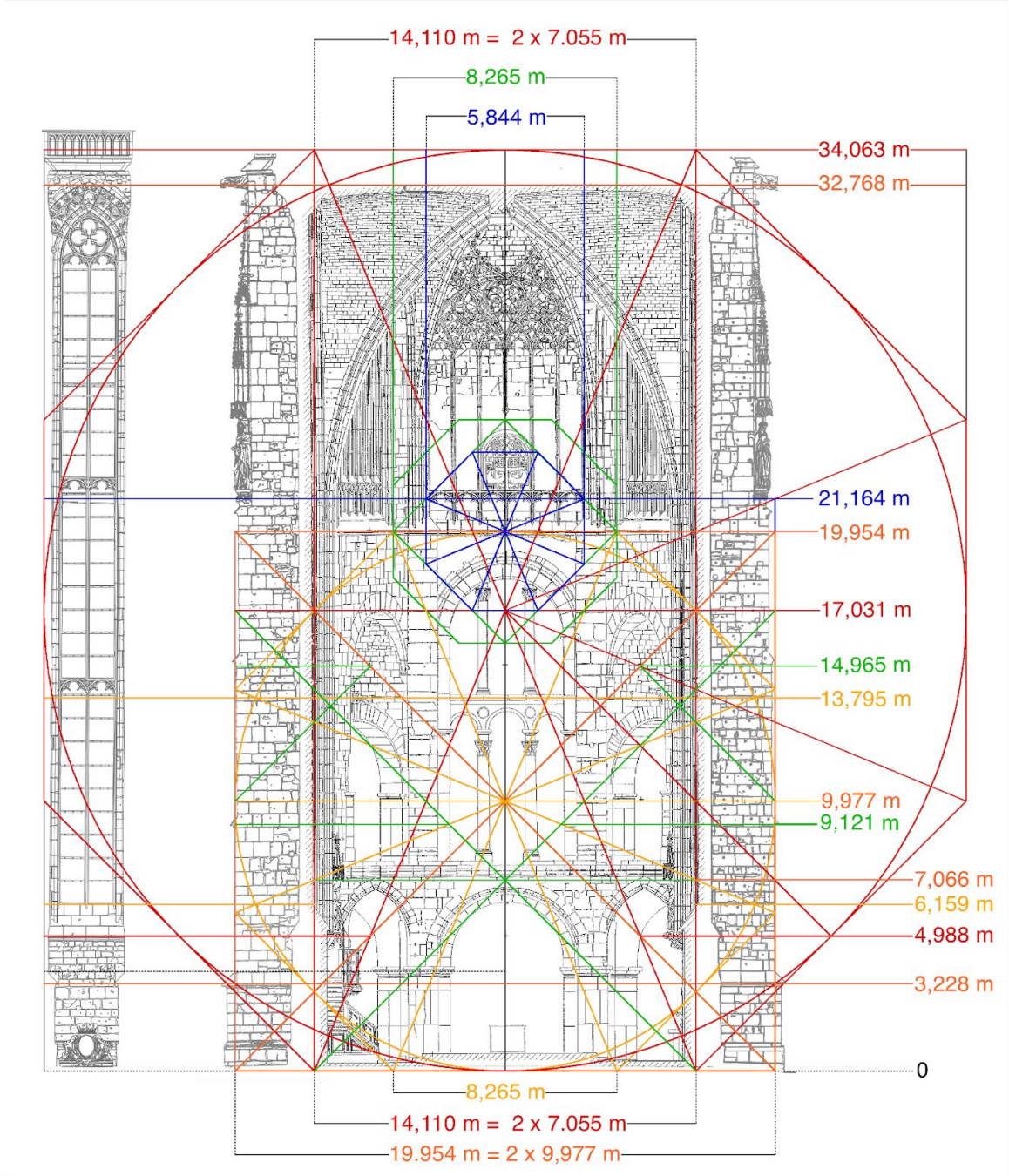

Figure 2
Aachen Cathedral, section and elevation of Gothic choir, with geometrical overlay

The blind west window (See Figure 3)

Blue lines
As mentioned previously, the blue octagon locked into the overall elevation system determines the total width of the window, which is 5.844m between mullion axes.

Red lines
A red square can be inscribed within the blue octagon, so that its top edge reaches height 22.020m, and a rotated red square can be inscribed within it in turn. The sides of this rotated red square cut the corner-to-corner radii of the blue octagon at height 21.415m, defining the top edge of the round-arched tracery arcade at the base of the window. An equilateral triangle placed at this level and filling the width of the window space has its apex at height 26.476m, the level where the six lancets of the main window begin to taper.

Orange lines:
The semicircular framing arch of the lancet tracery, which spans the full window, reaches a height of 29.399m. Two arcs of the same radius spring from the centerline of the window, working together with this arch to define the arches of secondary rank in the tracery pattern, each of which frames three of the lancets rising from below. A series of smaller arcs, each large enough to span four lancets, intersect with each other to locate the lancet tips at height 27.761m. When the central one of these arcs is completed into a circle, one finds the top of the console at height 24.528m, which supports the central tracery mullion above the window of the Carolingian octagon.

Yellow lines:
A sequence of operations marked in yellow establishes the springline and apex of the window head. The interval between the previously established heights 24.528m and 29.399 can be halved, as the crossing yellow lines indicate, to establish a new level at height 26.963m. This is the springline of the window head. Quarter circles with radii equal to the 5.844m span of the window can be placed at this level, such that their intersection point defines the apex of the window head. The window height thus defined agrees to a tolerance of less than 3cm with the vault height of 32.766m previously established with red lines in Figure 2. This agreement shows how carefully the window design was integrated into the overall design of the choir elevation.

Green lines:
The crowning element in the window tracery—a rotated square with convex sides—develops within a circle filling the space between the previously established heights 29.399m and 32.024m. A square and a rotated square can be inscribed within the circle, such that the corners of the square serve as midpoints of arcs that define the curving sides of the final rotated figure.

Blue lines:
When a slightly larger rotated square is inscribed between the heights 27.761m and 32.024m, its lower diagonal facets help to locate the baselines of the two curving tracery

panels that fill the space between the semicircular framing arch and the outer frame of the window zone. The heads of these panels have proportions set by the geometry of equilateral triangles.

All the constructions seen in the elevation work together to form a coherent geometrical network that appears to have been fully planned out before the start of work on the choir. All of these constructions, moreover, emerge directly out of the geometry of the ground plan.

In sum, the geometry of the Aachen choir follows that of the Sainte-Chapelle only indirectly. The Aachen choir exceeds that venerable 13th-century prototype in height, in accord with a taste for octagon-based elevation systems that was enjoying widespread popularity in the middle decades of the 14th century. It seems likely that the Aachen project may have been inspired in this respect by the choir of Prague Cathedral, whose construction was launched by Emperor Charles IV, and which also features an octagon-based elevation scheme. Such a visual reference would underscore the rank of Aachen as a crucial site of the empire in general, and as a focus of the cult of Charlemagne that Charles IV had done a great deal to promote. This scenario can be proposed only as a hypothesis, but the case of Aachen presented here at least suggests the value of integrating geometrical and technical analyses more fully into the still largely style-based methods with which Gothic buildings and their meanings have been studied.

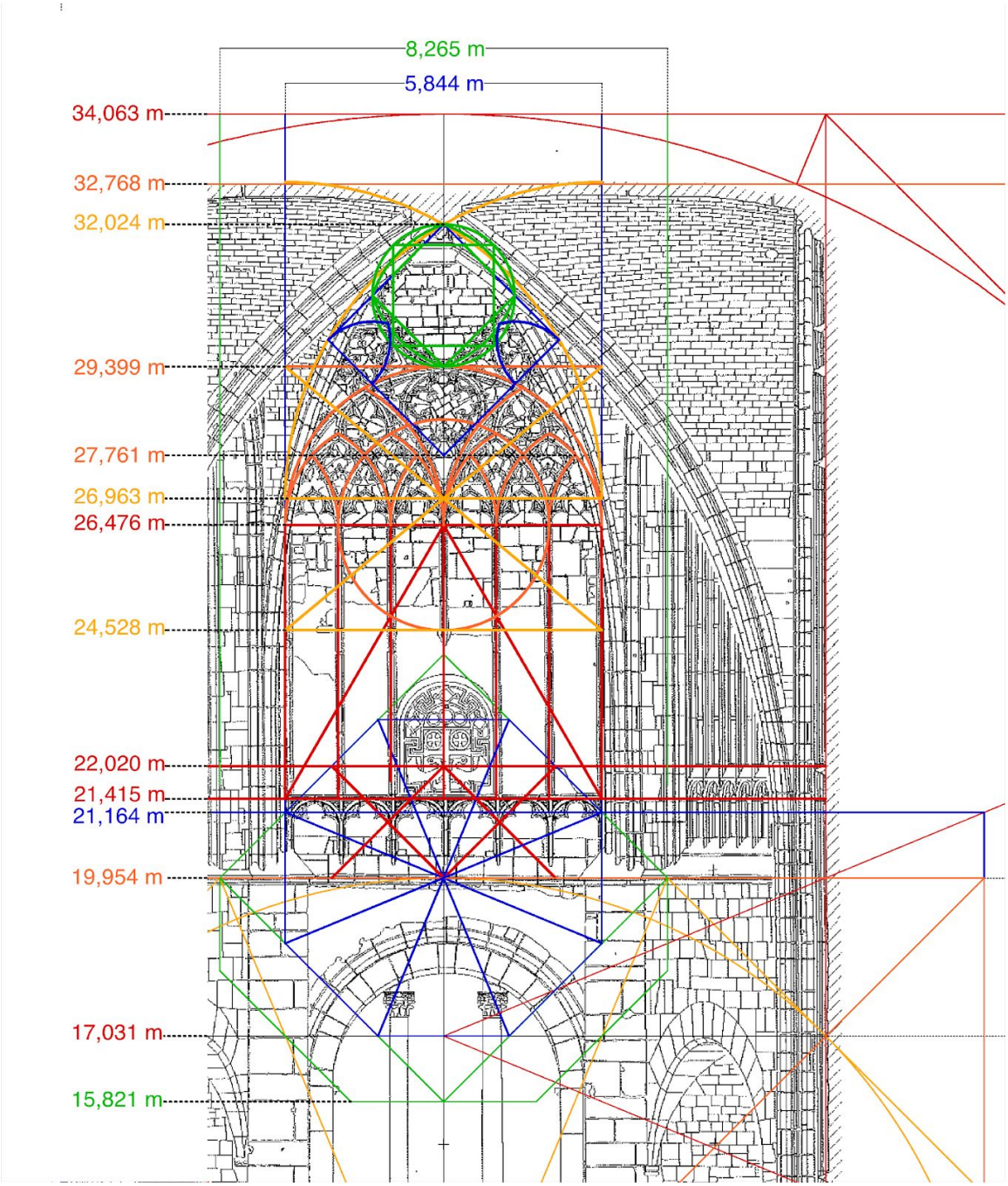

Figure 3
Aachen Cathedral, detail view of blind west window in choir