

Robert Bork

A Geometrical Investigation of the Cistercian Church at Altenberg¹

Introduction

The Cistercian church at Altenberg, like most Gothic buildings, appeals to the eye in large part because its forms seem to express a kind of geometrical order. This is true not only of details such as the tracery windows, but also in the overall plan and elevation of the building. As the following analysis will reveal, a surprisingly unified geometrical framework appears to govern the whole building, despite a series of important detail differences that distinguish the fourteenth-century work on the nave from the thirteenth-century work on the choir. For this reason, it is necessary to consider the geometry of the choir ground plan before moving on to explore the geometry of the choir and façade elevations.

The plan of the Altenberg choir involves interesting and highly original permutations on design themes seen in the choir of Cologne Cathedral. In both cases, there are seven apsidal chapels tightly packed around a single ambulatory. In Cologne, each chapel and the adjacent ambulatory bay occupies a wedge of exactly 30 degrees, or one twelfth of a circle. In Altenberg, the plan geometry is more subtle, and this 30-degree subdivision is not explicitly expressed. As the following paragraphs will reveal, however, many of the proportions of the Altenberg choir plan were actually set using this idea. To set a stage for this discussion, it is helpful to consider the simpler geometry of the Cologne choir in more detail.

The Ground Plan of the Cologne Cathedral Choir (See Figure 1)

The east end of Cologne Cathedral was built atop a perfectly circular semi-circular podium whose geometrical center corresponds to that of the apse. This semi-circle, and the lines dividing it into 30-degree slices, are shown in red in Figure 1. Within each half of the semi-circle, a red square can be inscribed; it is convenient to call the length of each square 1.000 unit, so that the radius of the semi-circle is $\sqrt{2}$, or 1.414. This relationship between squares and their circumscribing circles is closely associated with the nesting of rotated squares known as quadrature, which is well known in the literature on Gothic architecture. It is important to emphasize, though, that Gothic builders exploited the relationship between circles and other polygons, including octagons, hexagons, and, in cases like the Cologne choir, twelve-sided dodecagons. The fundamental principle was the same in every case. Each time one steps inward, the radius in question shrinks by a factor corresponding to the cosine of the angle in the polygon. In the case of a square, for example, the relevant ratio is $\cos(45^\circ) = .707 = 1/1.414$. In the case of a dodecagon, the ratio is $\cos(15^\circ) = .966$. If one steps inward by two such steps, then the radius

¹ This essay is adapted and excerpted from the author's contributions to Sabine Lepsky and Norbert Nußbaum, *Gotische Konstruktion und Baupraxis an der Zisterzienserkirche Altenberg, Bd. 2: Quer- und Langhaus* (Bergisch-Gladbach, 2012), pp. 74-88.

shrinks by a factor of $(.966)^2 = .933$. In the Cologne choir, this radius corresponds to the span from the apse center to the centers of the engaged piers between the chapels, as the green arc in Figure 1 shows. The medieval designers of the Cologne choir required no theoretical knowledge of cosines to establish this dimension. They simply had to construct arcs and polygon sides within the 30° slices constituting the ground plan. A third step inward yields a dimension of $(.966)^3$, or .901, which marks the distance from the apse center to the arches separating the chapels from the ambulatory, as the green line segments in Figure 1 show. A fourth such step gives a dimension of $(.966)^4$, or .870. Half of this dimension, or .435 units, is the distance between the centerline of the Cologne choir and the centerpoints of the piers in its straight bays. These relationships are shown in blue in Figure 1, along with the closely associated construction of the free-standing piers' center points. It is striking that the relative proportions of the Cologne choir composition can be so readily established using only simple steps based on the relationship between dodecagons and their circumscribing circles.

The Ground Plan of the Altenberg Choir (see Figures 2, 3 and 4)

The designer of the Altenberg choir evidently followed many of the same basic steps, although he later went on to add novel ideas of his own. At Altenberg, a fundamental dimension appears to have been the span between the outer corners of the buttresses separating the straight bays from the chapels. In Figure 2, this span serves as the diameter of a red semi-circle around the chapels, analogous to the podium-defining arc seen at Cologne. At Altenberg, the radius of this semi-circle is 15.462 meters. Red diagonals connecting the base of the semi-circle to its easternmost point align with the walls of the easternmost chapel. It may be significant in this context that these chapel walls and the buttresses at the ends of the semi-circle appear on archaeological grounds to be among the oldest parts of the choir. Just as in Cologne, many relevant distances can be found by stepping in through polygon rotation. So, for example, the inner surface of the choir walls stand 13.931m from the building centerline, a distance that is three dodecagon steps in from the framing circle; $15.462\text{m} \times (\cos 15^\circ)^3 = 15.462\text{m} \times (.966)^3 = 15.462\text{m} \times .901 = 13.931\text{m}$. In a similar vein, the radius from the apse center to the engaged columns between the chapels can be found by stepping in from the red semi-circle first by a factor of $\sqrt{2}$, and then by a factor of $(\cos 15^\circ)^2$. This means that the engaged columns lie on a circle of radius $15.462\text{m} \times .707 \times .933 = 10.201\text{m}$. This circle, and the 15-degree steps inward, are shown in green in Figure 2.

As the blue portions of Figure 2 show, the size of the hemicycle at Altenberg was found slightly differently than at Cologne. The radius from the apse center to the centers of the free-standing piers at Altenberg is 5.467m, or .500 units. The radius of the hemicycle is thus smaller than the radius to the outer buttress corner by a simple factor of $1/2\sqrt{2}$. The distance from the building centerline to the axes of the piers in the straight choir bays is smaller by another factor of $\cos(15^\circ)$, so that it equals $5.467\text{m} \times .966 = 5.281\text{m}$. This dimension, which derives simply from the geometry of the choir ground plan, would go on to govern the elevation of both the 13th-century choir and the 14th-century nave, as subsequent discussion will demonstrate. Before going on to consider these elevations,

though, it is appropriate to consider the rest of the choir ground plan, which introduces several new ideas quite unlike those seen at Cologne.

As Lepsky and Nußbaum discovered, the distances between the free-standing piers of the Altenberg hemicycle appear to have been set using standardized modules based on the pier diameter.² In particular, each pier was found to have a diameter of 83cm, and the spaces between their adjacent faces was found to be 2.5 times as great, so that the distance between their centers would be $3.5 \times (.83\text{m}) = 2.905\text{m}$. These intervals are shown in Figure 3 by the small orange semi-circles that cluster along the arc of the hemicycle like bead on a necklace. The five eastern facets of the hemicycle are effectively identical, but the two western facets differ very subtly from the others because the western pier centers lie on the previously established axes of the straight bays, 5.281m from the building axis, rather than on the arc of the hemicycle. The two larger orange arcs at the base of the hemicycle in Figure 3 show how the 2.905m interval can be swung down onto these axes to locate these pier centers. An orange horizontal through those pier centers marks the divide between the rectilinear geometry of the straight bays and the radial geometry of the chevet; this line is 1.703m to the west of the chevet's geometrical center.

The westernmost engaged piers of the ambulatory fall where this orange horizontal intersects the previously established arc of radius .933 units, i.e. 10.201m. The two dotted orange lines connecting these pier centers to the chevet center subtend an arc measuring 198.96° . Subdivision of that arc into seven equal pieces gives theoretical chapel openings of 28.422° measured between the centers of the engaged piers, which are highlighted by small orange circles in Figure 3.

It is interesting and significant that the actual chapel openings at Altenberg are slightly uneven in size, with the western chapel entrances measuring about 5cm narrower than the other five. This anomaly hints at a geometrical and conceptual structure not seen in the more strictly regular Cologne chevet. The new ingredient at Altenberg appears to have been a kind of scenographic thinking, as Lepsky and Nußbaum have argued previously.³ From the point of view of a spectator standing at the high altar, the engaged columns framing the chapel openings disappear behind the free-standing columns of the hemicycle.⁴ This is actually a somewhat surprising result, for two reasons: first, because the optical center defined by the altar is roughly a meter west of the geometrical center of the chevet composition; and second, because the spacing between the free-standing piers

² Sabine Lepsky and Norbert Nußbaum, *Gotische Konstruktion und Baupraxis an der Zisterzienserkirche Altenberg, Bd. 1: Die Choranlage* (Bergisch-Gladbach, 2005), pp. 55-63.

³ Lepsky and Nußbaum (2005), pp. 46-47.

⁴ Robert Bork, "Ground Plan Geometries in Suger's St-Denis: A Prototype for Altenberg," in Astrid Lang and Julian Jachmann, eds. *Aufmass und Diskurs* (Cologne: University of Cologne, 2013), 55-68.

appears to have been set using a simple modular recipe, rather than precise geometrical subdivision of the hemicycle arc.

Figure 4 shows how these distinct compositional systems could have been combined to give the visually satisfying but slightly irregular engaged pier spacings seen at Altenberg. As a starting point, one can take the theoretically determined 28.422° span between the engaged piers framing the eastern axial chapel. The distance between these centers would be 5.06m, which agrees superbly with the 5.03m distance measured in the building. From these outer pier centers, solid purple lines can be projected inwards through the corresponding free-standing piers; these lines converge 1.10m west of the chevet's geometrical center. This convergence point on the altar table can then become an optical center from which four dotted purple lines can be projected outwards through the free-standing pier centers to locate the adjusted locations of the four intermediate engaged piers. This construction produces the following interaxial chapel openings, read from north to south:

4.953m 5.047m 5.073m 5.060m 5.073m 5.047m 4.953m

These figures agree closely with the dimensions found in the building:

4.97m 5.05m 5.03m 5.03m 5.02m 5.015m 4.975m

The logic of the optical-scenographic construction described here is admittedly somewhat convoluted, and its steps must be considered more hypothetical than the very clear steps used to generate the larger elements of the choir ground plan. It is striking, nevertheless, that this proposed construction effectively reproduces the irregular pattern of pier spacing seen at Altenberg, which neither a simple modular recipe nor the regular geometrical structures of the Cologne type can. Taken together, the steps of this analysis thus help to clarify both the originality of the Altenberg choir designer, and his relationship to the broader currents of Gothic design.

The elevation of the Altenberg choir (see Figure 5)

In Gothic building practice the proportions of the elevation were typically developed from those of the ground plan. This can be seen especially clearly in tower designs, such as the famous drawing of the Laon Cathedral towers made by Villard de Honnecourt, or the slightly later but more precisely executed façade drawings for the cathedrals of Strasbourg and Cologne. In Altenberg, analogous procedures appear to have been used to establish the shape of the choir vessel, since the dodecagon seen in the ground plan generates dimensions also seen in the elevation. The floor level of the choir serves as the geometrical baseline for all such constructions, as shown in Figure 5.

A dodecagon centered on the middle of this baseline and with a radius of 15.462m rises to define the drip molding at base of the clerestory window sill, which plays an important role in the organization of the exterior elevation. Since the dodecagon is framed by the outer margins of the straight bay buttresses, the width and elevation of the choir are integrated through this single proportioning figure. Further levels of the exterior also result from this figure: the drip molding on the top edge of the aisle coincides with another corner of the dodecagon, 11.319m above the inner choir floor, and the drip

molding beneath the aisle window sill can be found by at a height of 2.072m, corresponding to a quarter of a dodecagon facet. The radial line to the corner of the dodecagon intersects the circle of radius 10.934m defined in the ground plan, moreover, at a height 2.830m, thus defining the top edge of the aisle window sill. The constructions seen in the ground plan thus help to establish important heights relevant for both the exterior and the interior articulation. These constructions, however, do not suffice to fully determine the elevation. Instead, the key figure in the choir section is a single great octagon, whose lower facet coincides with the floor of the central vessel. Similar great octagons of this type also appear to have governed the proportions of the cathedrals of Clermont-Ferrand and Prague, begun in 1248 and 1344, respectively.

As mentioned previously, the northern and southern arcade axes at Altenberg stand 10.562m apart, so that the span from the building centerline to the pier centerline is 5.281m, in accord with the regular dodecahedral construction shown in Figure 2. The great octagon in the choir elevation is 2.414 times larger than the choir span, so that it reaches an overall height of 25.496m, as shown in Figure 5. This height corresponds both to the building's vault height at the keystone, and to the level where a prominent molding steps outward to terminate the gutteral walls. The equator of the octagon, at a height of 1.207 choir spans or 12.748m, corresponds quite precisely to the base of the openings in the triforium. And the lower lateral corners of the octagon, at height .707 or 7.468m, coincide closely with the tops of the main arcade capitals. All of these relationships appear in red in Figure 5.

To go further in the analysis of the choir elevation, it necessary to move beyond the basic structure of the governing octagon. The tops of the capitals in the main vault, for example, fall at a height of 1.669 or 17.678m, which is the level where a circle inscribed within the octagon intersects the rays reaching to its upper lateral corners. The openings in the triforium terminate at height 1.414, or 14.935m, where those same rays intersect the vertical axes of the main piers. The thickness of the wall equals half the height of the openings, as the green 45-45-90 right triangle between these levels indicates. The similar green triangles immediately below show that the distance from the triforium baseline to the keystones of the arcade arches equals the height of the triforium openings. The outer plane of the wall intersects the red ray of the octagon at height 1.457 or 15.388m, the level where a horizontal lip molding steps out from the outer wall, capping the triforium zone and the sloping roof that leans up against it. The drip molding just above, as noted previously, falls at height 15.462m. The baseline of the clerestory falls just a bit further up, at a height of 1.509 or 15.935 units. The interval between the base of the lip molding and the clerestory baseline is one half of the wall thickness, as small blue 45-45-90 right triangle in this zone indicates.

Further details can also be developed within this framework, but the preceding discussion suffices to show the basic relationship between the elevation of the Altenberg choir and its octagon-based geometrical armature.

Naturally it would have been impractical to construct a 25-meter high octagon at the building site, and to take measurements directly from it. Instead, the builders seem to have used arithmetical approximation techniques to the geometrically defined quantities, much as they did in the layout of the choir groundplan and its columns. In that instance, the interval between the hemicycle columns was set as a simple multiple of their 83cm diameter. Lepsky and Nußbaum previously established that the arcade columns of the choir were nine times as high as they were thick, and that the triforium baseline, high vault capitals, and high vault keystones lie 15, 21, and 30 column diameters above the choir floor, respectively.⁵ Clearly, however, there were other geometrically defined quantities, such as the clerestory wall thickness, that could not be easily translated into such round numbers.

In this byplay between geometrical and arithmetical procedures, one begins to see how Gothic builders brought their formal ideas from conception, through the drawing stage, and to the final realization on the worksite. It is striking how simple geometrical constructions can determine the outlines of a complex chevet structure like that of Altenberg, whose design was clearly informed by but distinct from that of Cologne Cathedral. As a practical matter, since drawing the entire scheme at full scale would have been effectively impossible, it clearly made good sense to approximate key dimensions in the design drawings as multiples of a standardized unit of measure. This arithmetical approximation to geometrically determined quantities should be distinguished from the search for harmonic modular intervals seen in Renaissance architectural practice, which was inspired by Pythagorean musical theory. Franklin Toker thus rightly observed that this medieval design practice was not truly module-based, but only “pseudo-modular.”⁶ How long after the completion of the choir did the Altenberg workshop hold to this method?

For the ground plans in the second building campaign, which involved the transept and the nave, true modularity rather than geometry appears to have played the dominant role, but for the design of the nave elevation and the west façade, which was designed a century and a half after the choir, geometry remained crucial. In the nave elevation, one can see the clever and pragmatic ways in which the designers adapted the reference levels established in the earlier work on the choir, while integrating them into a substantially new architectural system. The design of the west façade, meanwhile, shows that the great octagon used to establish the cross-section of the choir had continuing relevance over the long history of the building project, even as it, too, was reinterpreted.

The geometry of the Altenberg nave elevation (see Figures 6 and 7)

In the nave, several main levels inherited from the choir serve as the geometrical anchors of a new and subtly altered proportional scheme. The overall heights of the aisle and

⁵ Lepsky and Norbert Nußbaum (2005), pp. 59-63.

⁶ Franklin Toker, “Gothic Architecture by Remote Control: An Illustrated Building Contract of 1340,” *Art Bulletin* 67: 67–95.

clerestory walls, for example, had to be respected in order to insure the formal coherence of the whole edifice. The manner in which this was done can clearly be seen in the exterior elevation of the nave.

As a starting point, it is helpful to consider the relationship between the aisle windows and the buttresses that flank them, as seen in Figure 6. The red square has a side length of 5.03m, equal to the full length of a nave bay. The top of the square was set at height 7.468m, even with the bottom of the first buttress setback; this reference level was already established in the choir as the corner of the great red octagon seen in Figure 5. The lower edge of the red square gives the baseline of the aisle window sill. The length of the bay, which Lepsky and Nußbaum have convincingly interpreted as part of the new module-based planning of the nave as a whole, thus helps to determine the aisle elevation in a way that depends on the geometry initially established in the choir.⁷ This shows the high priority that was given to the linking of the new work with the old. When one uses quadrature to construct an red circle and an orange square within the red framing square, one finds the width of the aisle windows measured between the axes of their outer mullions, which is $5.03\text{m}/\sqrt{2} = 5.03\text{m} \times .707 = 3.56\text{m}$.

Orange verticals rising through these mullion axes intersect the top of the original red square in points that serve as the centers of curvature for the aisle window heads, which have the proportions of an equilateral triangle, as the green lines in Figure 6 indicate. Analogous arcs with the same centers but spanning all the way to the buttress centerlines intersect 3.908m above the top of the red squares, or 11.376m above the choir floor, where $11.376\text{m} = 3.908\text{m} + 7.468\text{m}$. This height marks the lip of the drip molding terminating the aisle wall, and it comes very close to the height of 11.319m established in the choir using the dodecagon construction, although the relationship is not mathematically precise. The drip molding thus steps up very slightly from the east end into the nave.

The height of the drip molding also plays a role in determining the free span between the buttress margins. Blue diagonals dropping from the bay divisions at the level of the drip molding intersect with the red diagonals of the original red generating square at points aligned with the horizontal iron bar separating the traceried head of the aisle window from the straight lancet sections below. These intersection points lie 3.91m apart, precisely on the margins of the buttresses. The width of each buttress is thus $5.03\text{m} - 3.91\text{m} = 1.12\text{m}$. The lower intersections of the buttress surfaces with the red diagonals, finally, align with the top edge of the aisle window sills, i.e. with the baseline of the window proper. All of the dimensions of the aisle wall, and of the associated windows, buttresses, and moldings, are thus developed directly out of a generating square whose positions was set by the buttress setback inherited from the choir.

The format of the clerestory wall recapitulates that of the aisle wall, but with slight variations. To understand this, it is helpful to insert two pairs of violet verticals, as seen

⁷ Lepsky and Nußbaum (2012), pp. 81-86.

in the righthand bay of Figure 6. The first of these lines rise halfway between the previously established orange and blue verticals, which were 3.56m and 3.91m apart, respectively; the first new violet lines are thus 3.73m apart. The second new lines stand as far to the outside of the blue lines as the first set does to the inside; in other words, they stand 4.08m apart, where $4.08\text{m} - 3.91\text{m} = 3.91\text{m} - 3.73\text{m} = .17\text{m}$, accounting for rounding.

Figure 7 shows the development of the clerestory zone within this framework. First, as seen in the middle left of the figure, the red square of bay length 5.03m is stacked on top of the baseline defined by the drip molding at height 11.376m. The red diagonals of this square cut the blue verticals framing the buttresses at a height that locates the drop molding at the base of the clerestory sill. This level can serve as the baseline for a second red square of side length 5.03, which will be important for the further development of the clerestory geometry. The top of the first square, meanwhile, defines the top edge of the clerestory window sill.

In the middle bay of Figure 7, an orange square of side length 3.56 has been stacked atop this sill; its upper edge locates the horizontal iron reinforcement bar that separates the traceried window head from the straight lancets below. These lancets are gathered into pairs, each surmounted by pointed arches whose geometrical centers coincide with the points where the diagonals of the orange square cut the violet verticals 3.73m apart. The clerestory windows are thus very slightly wider than those of the aisle, with a notional width of 3.73m rather than 3.56m.

The framing arches over the clerestory windows can readily be found using centers located where the top edge of the second red square cuts the violet verticals 3.73m apart; these points are picked out with small concentric circles in the upper right portion of Figure 7. Larger arcs tangent to the tracery mullions then define the clerestory archivolt. From these centers, also, red quarter-circles reaching all the way to the bay divisions rise up to uppermost drip molding on the clerestory wall, whose overall height was defined by the great octagon introduced in the construction of the 13th-century choir. This alignment cannot be dismissed as coincidental or merely pragmatic, since the design of the 14th-century west façade clearly depends on this same proportioning figure.

The geometry of the Altenberg west façade (see Figures 8 and 9)

The west façade at Altenberg relates very closely to the choir in geometrical terms. This is a surprising result, for several reasons. First, there is the simple fact that well over a century separated the conception of the choir from the construction of the west façade. Many modifications to the church design were introduced during this long interval, affecting everything from tracery forms and bay lengths to the units of measure used in the Altenberg workshop—the unit of 0.3320 centimeters seen in the choir columns was replaced by one of 0.3355 in the structures west of the crossing. And, while the basic widths of the aisles established in the choir were respected in the construction of the nave, slight shifts of axis appear in the outer buttresses of the façade. Even the floor level

in the church changes, since the floor in the nave is several steps lower than that of the choir. Every one of these factors would seem to argue against strong continuity of geometrical planning from the choir to the façade. And yet, the geometrical structure of the façade design closely echoes that of the choir.

The crucial geometrical element in the Altenberg west façade elevation is again a great octagon of the same size seen in the choir. Since the inner buttress axes of the façade align with the pier axes established in the choir, the corners of the great octagon lie on the buttress axes, 5.281m from the building centerline, as Figure 8 shows. The width of the octagon also matches the width of the lateral façade buttresses at ground level, although this relationship is only clearly visible on the north side of the façade, because the cloister structure intrudes on the south side. In the vertical dimension, meanwhile, the top facet of the octagon in the figure aligns with molding along the top edge of the façade, while its lower facet aligns with the sockel molding flanking the central portal. It might at first appear strange that this level, rather than the ground level, should be taken as the geometrical baseline of the façade composition. But, in fact, this is the same exact level as the floor of the choir. The sockel molding thus serves as a visual reminder of the building's architectural order, which remains coherent even as the ground of the surrounding site drops slowly from east to west.

The ordering power of the octagon starts to become apparent in the way that it governs the overall shape of the façade's great western window. The yellow-shaded triangles in Figure 8 correspond precisely to those shown in the choir in Figure 5; they are framed by the verticals connecting the octagon corners, and by the sloping lines from the octagon center to its upper lateral corners. A green circle concentric with the octagon and struck through the corner points of those triangles at height 1.414 or 14.935m intersects the diagonals of the octagon at height 1.590, or 16.789m. These points of intersection serve as the centers of two larger green circles that together define the extrados of the western window; these circles are just large enough to fit within the frame defined by the facets of the great octagon.

Figure 9 shows how derivatives of the octagonal geometry also govern the placement and thickness of the Altenberg façade buttresses and the subdivision of the western window. The green circles just described intersect the diagonals of the octagon at points .996 units or 10.198m from the building centerline. Blue verticals descending from these intersection points describe the axes of the outer buttresses. So, while these dimensions do not appear explicitly in the geometry of the choir, their derivation shown the ongoing importance of the octagonal geometrical frame seen in the choir elevation. The fact that the small gablets on the outer buttresses terminate precisely on the octagon's equator further underscores this point. The thickness of the forward-facing buttresses is equal to half the height of the yellow shaded wedges, as the two small pairs of rotated blue squares sitting on the octagon equator indicate. This means that the inner and outer buttress faces are .448 and .552 units out from the building centerline, which amounts to 4.735m and 5.827m respectively. The buttresses should thus be 109cm thick, which agrees well with the 110cm thickness measured on the building at window level; the

buttresses swell very slightly at ground level, attaining a total thickness of 113cm, but the schematic logic of the design seems fairly clear nevertheless.

The orange lines and shaded areas in Figure 9 describe the geometry of the western window. The width of the window was already in Figure 8 set by the establishment of the geometrical center points at height 1.590 or 16.789m. These points establish the baseline of the shaded equilateral triangle in Figure 9, whose upper vertex corresponds to the top of the window head. The orange arcs struck between its corners correspond to the intrados of the arch framing the window. It is interesting and significant that the centers of curvature used to define the window stand on the geometrical centers of the mullions framing the window. This means that the interval between the centers can be readily divided into eight slices corresponding to the lancets of the windows. This same interval sets the height of the lancets beneath their traceried heads, so that their straight portions fill a perfect square between heights 1.393 and .627, i.e. between heights 14.709m and 6.626m. It makes sense to think of the square as hanging down from height 1.393, since the location of the iron reinforcing bar at this level marks the intersection between the inner buttress faces and the top edges of the yellow-shaded triangles that relate to the geometry of the great governing octagon. Similar stacks of arcs and polygons appear in the tracery for the smaller windows in the church at Altenberg, as Lepskys and Nußbaum have shown.

There is obviously much more that can be said about the geometry of the Altenberg façade, and about the geometry of the Altenberg nave zone more generally. Even the very short preceding discussion, though, suffices to demonstrate remarkable methodological continuity within the Altenberg workshop. The ground plan of the church's thirteenth-century choir relates closely to that of Cologne Cathedral, while introducing modular and optical planning methods not seen in the more strictly geometrical composition of the metropolitan church. The elevation of the Altenberg choir was set by a single great octagon whose facet length derives from the choir span set in the ground plan. And this same octagon, placed at the same absolute height, would go on to govern the composition of the church's fourteenth-century façade, despite the drop-off of the ground level in between and the slow pace of work in the interval, which saw the introduction of new units of measure into the Altenberg workshop. On this basis it seems clear that geometry had a certain conceptual primacy in Gothic design, although modular approximations to geometrical figures were evidently of high importance to the workers actually involved in construction.

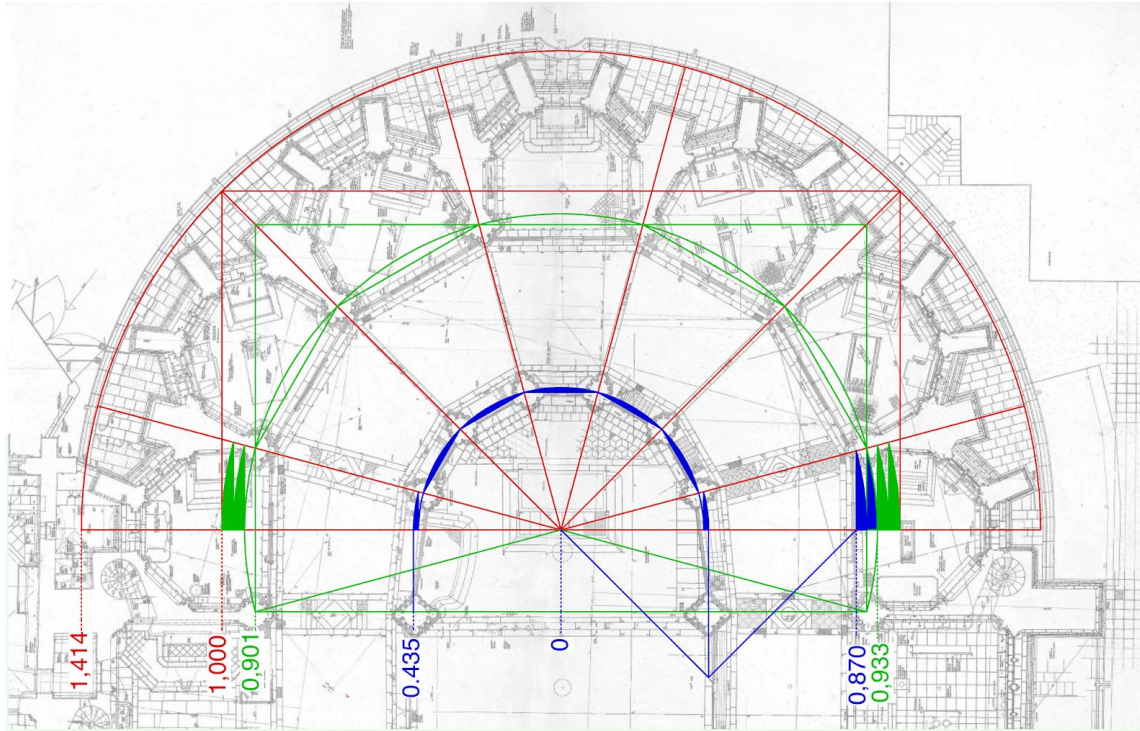


Figure 1: Cologne Cathedral, chevet plan with geometrical overlay.

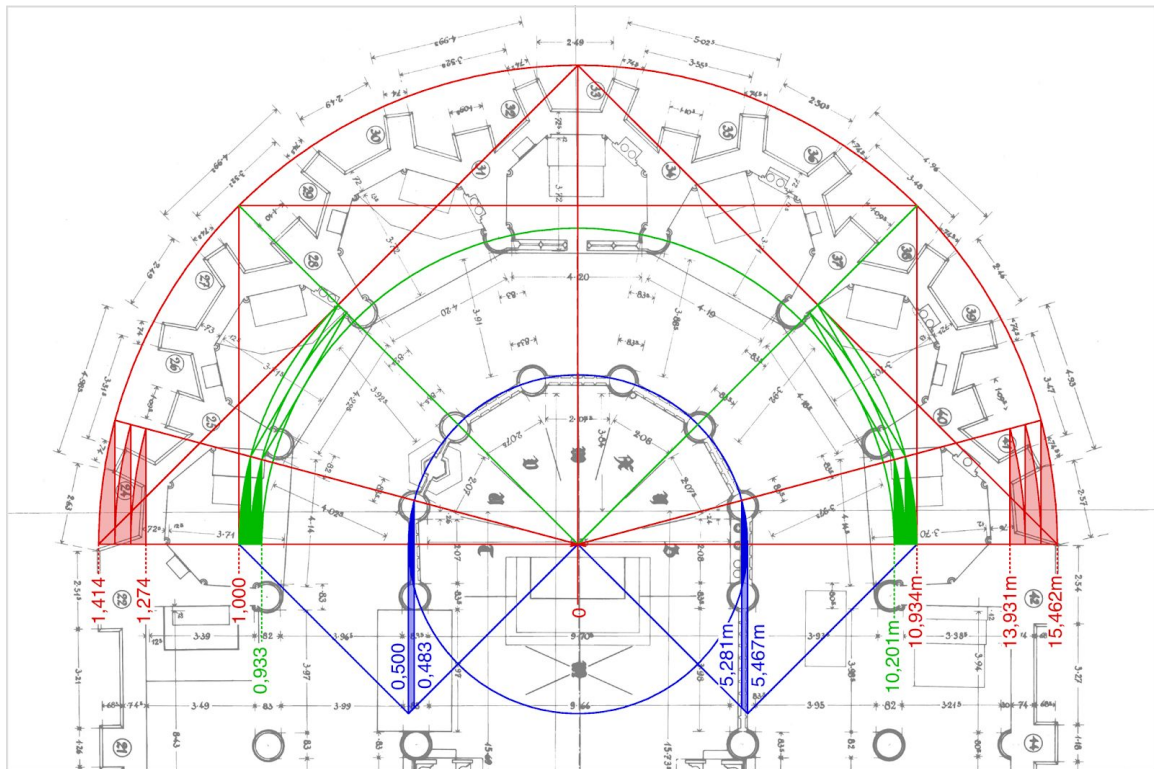


Figure 2: Altenberg, Cistercian Church, chevet plan with geometrical overlay, stage 1.

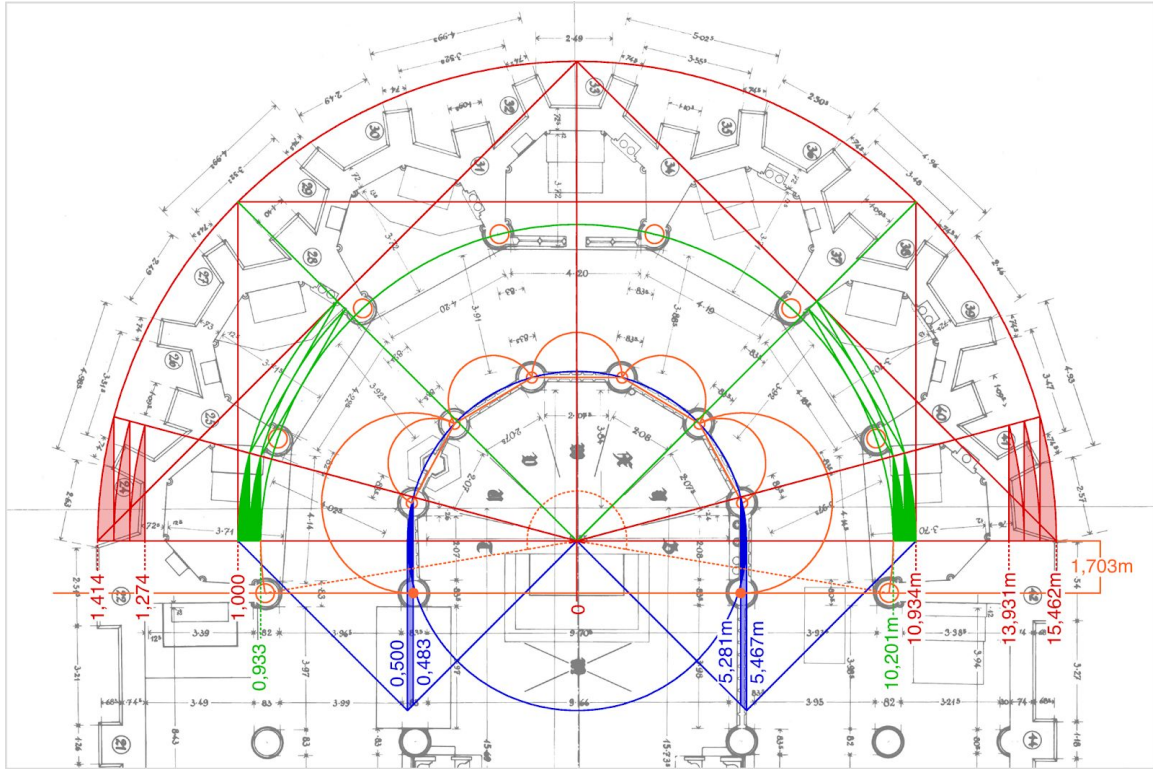


Figure 3: Altenberg, Cistercian Church, chevet plan with geometrical overlay, stage 2.

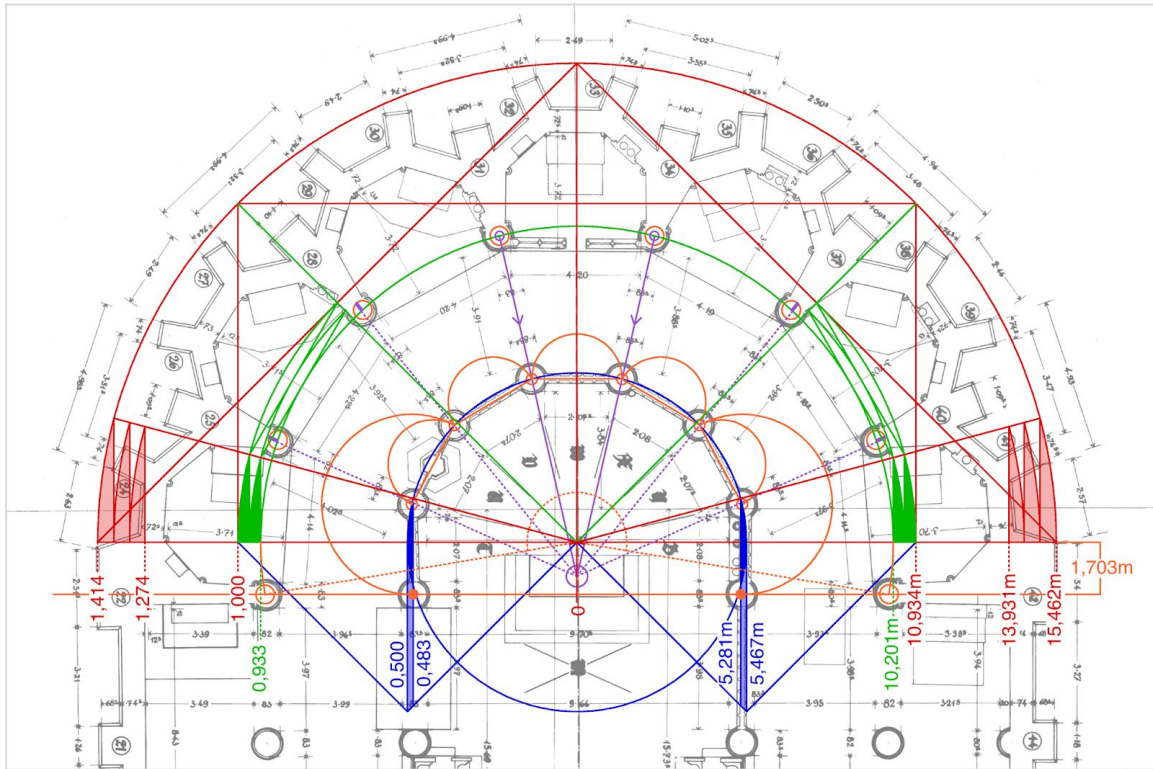


Figure 4: Altenberg, Cistercian Church, chevet plan with geometrical overlay, stage 3.

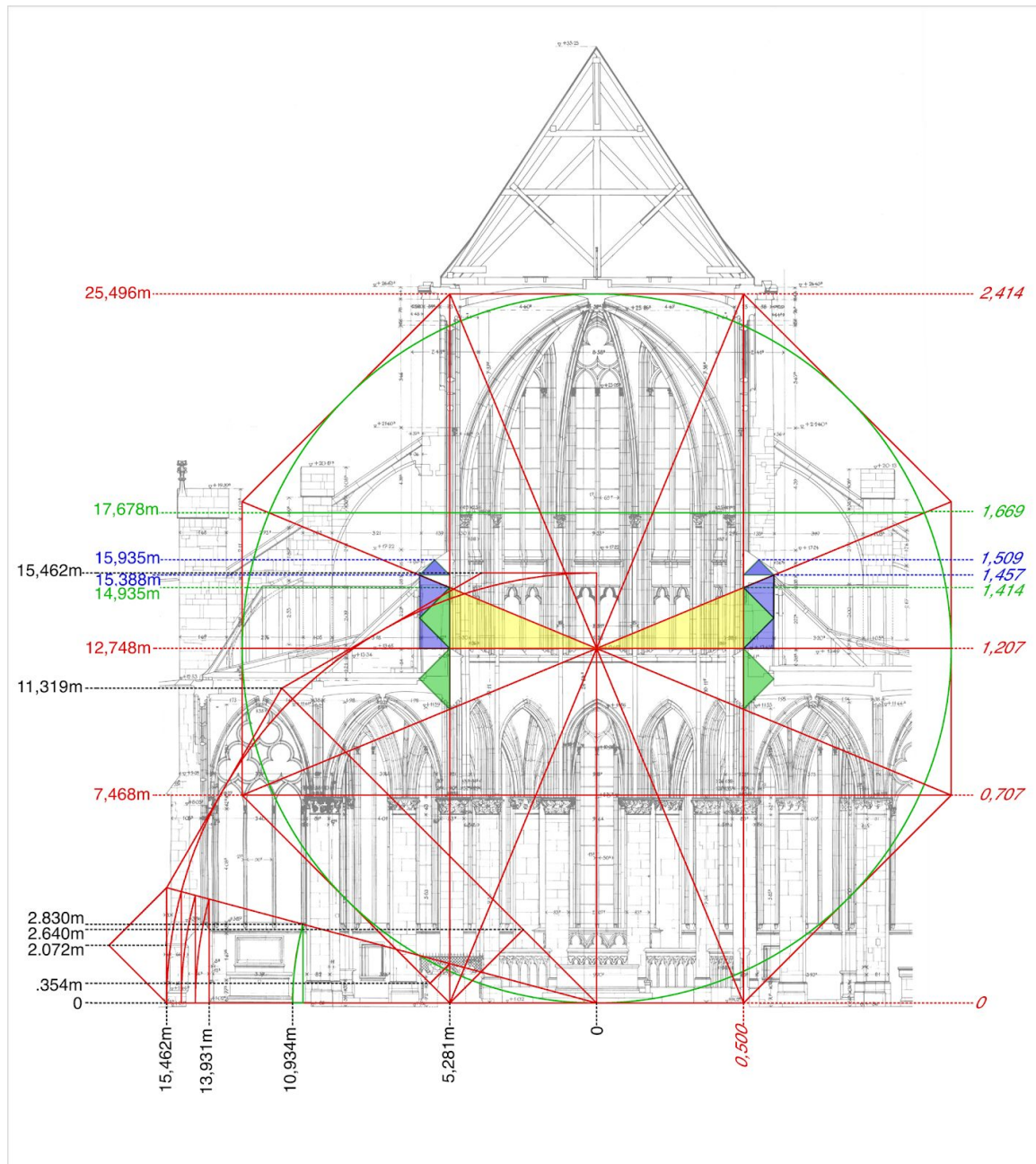


Figure 5: Altenberg, Cistercian Church, choir section with geometrical overlay.

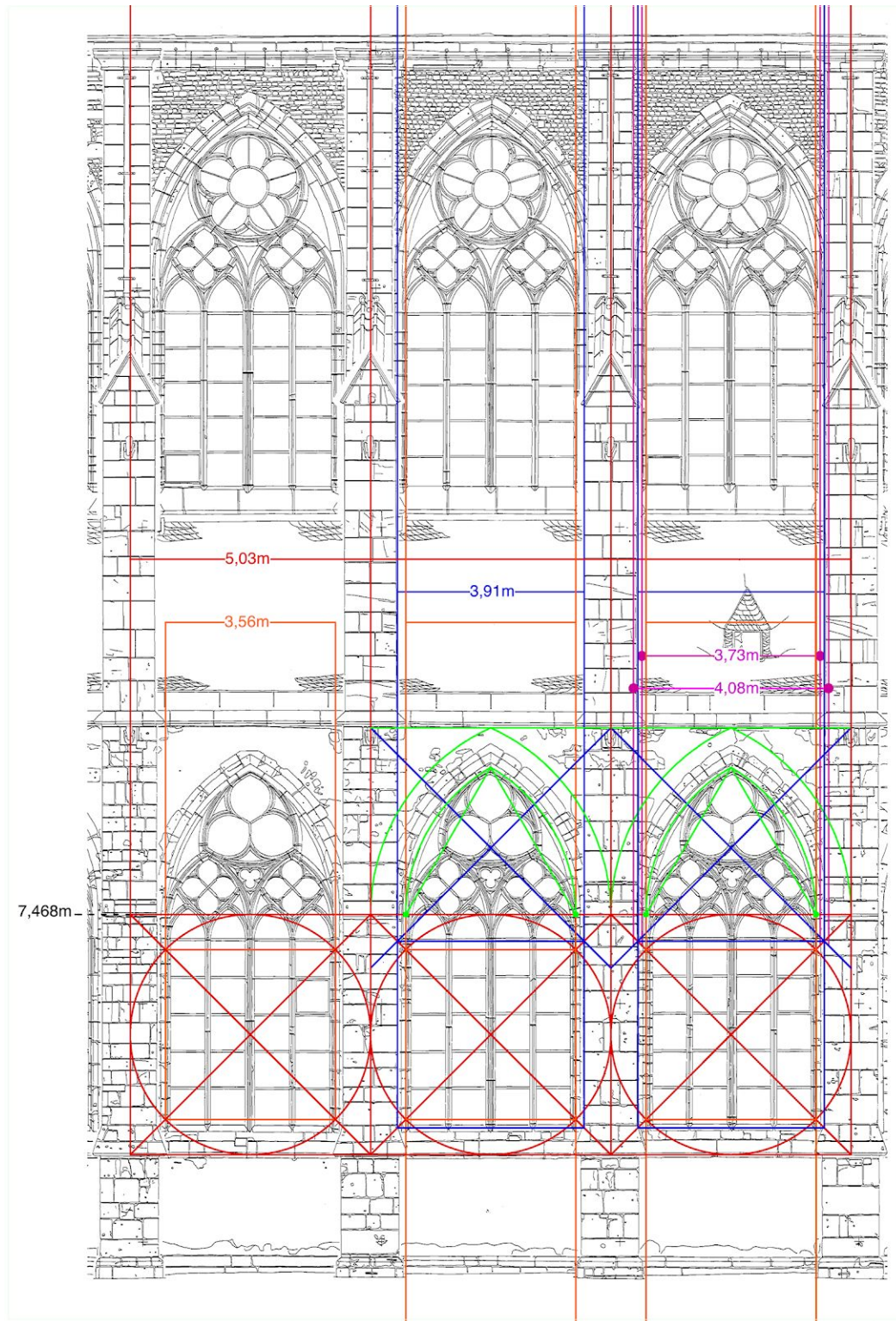


Figure 6: Altenberg, Cistercian Church, nave wall with geometrical overlay, stage 1.

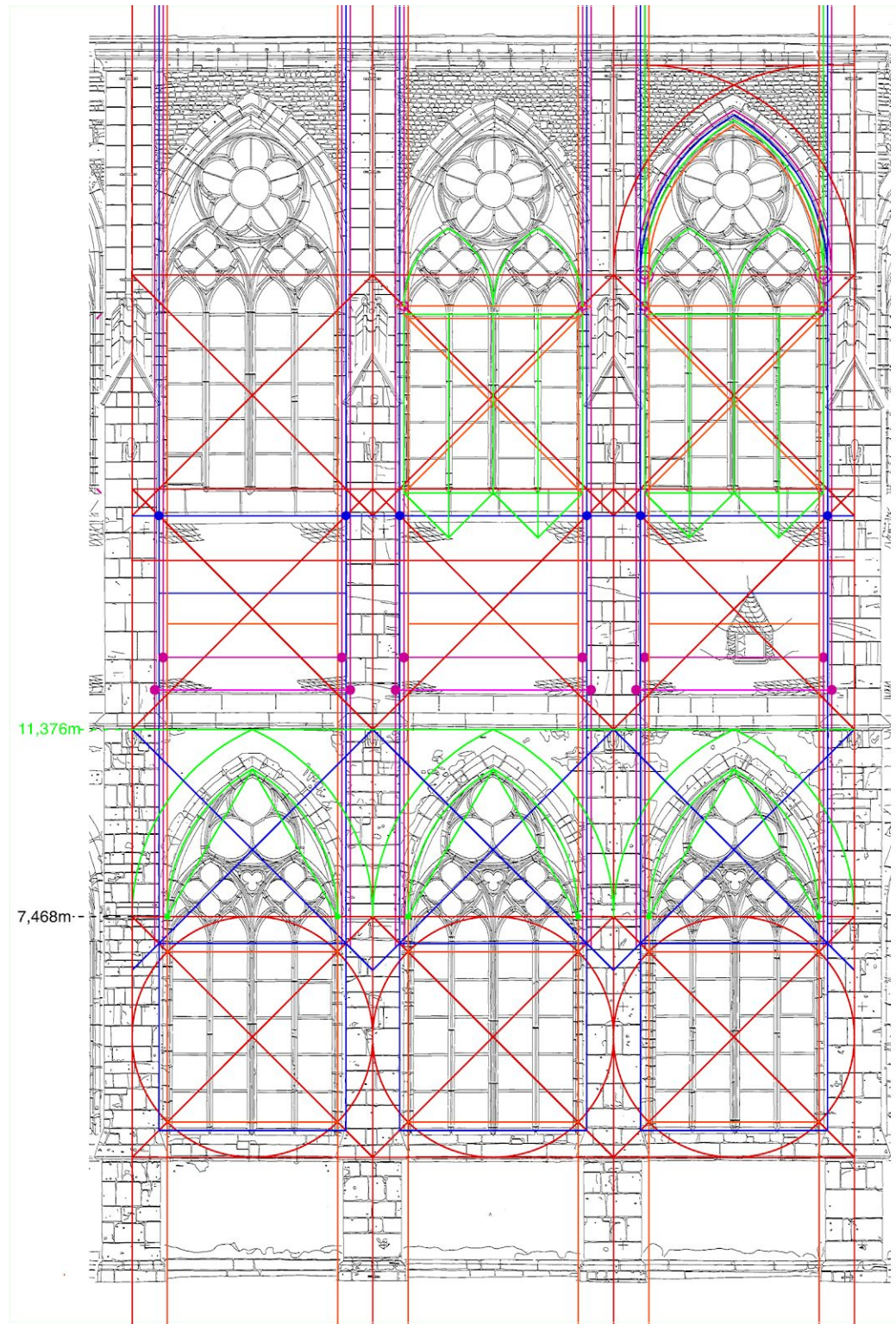


Figure 7: Altenberg, Cistercian Church, nave wall with geometrical overlay, stage 2.

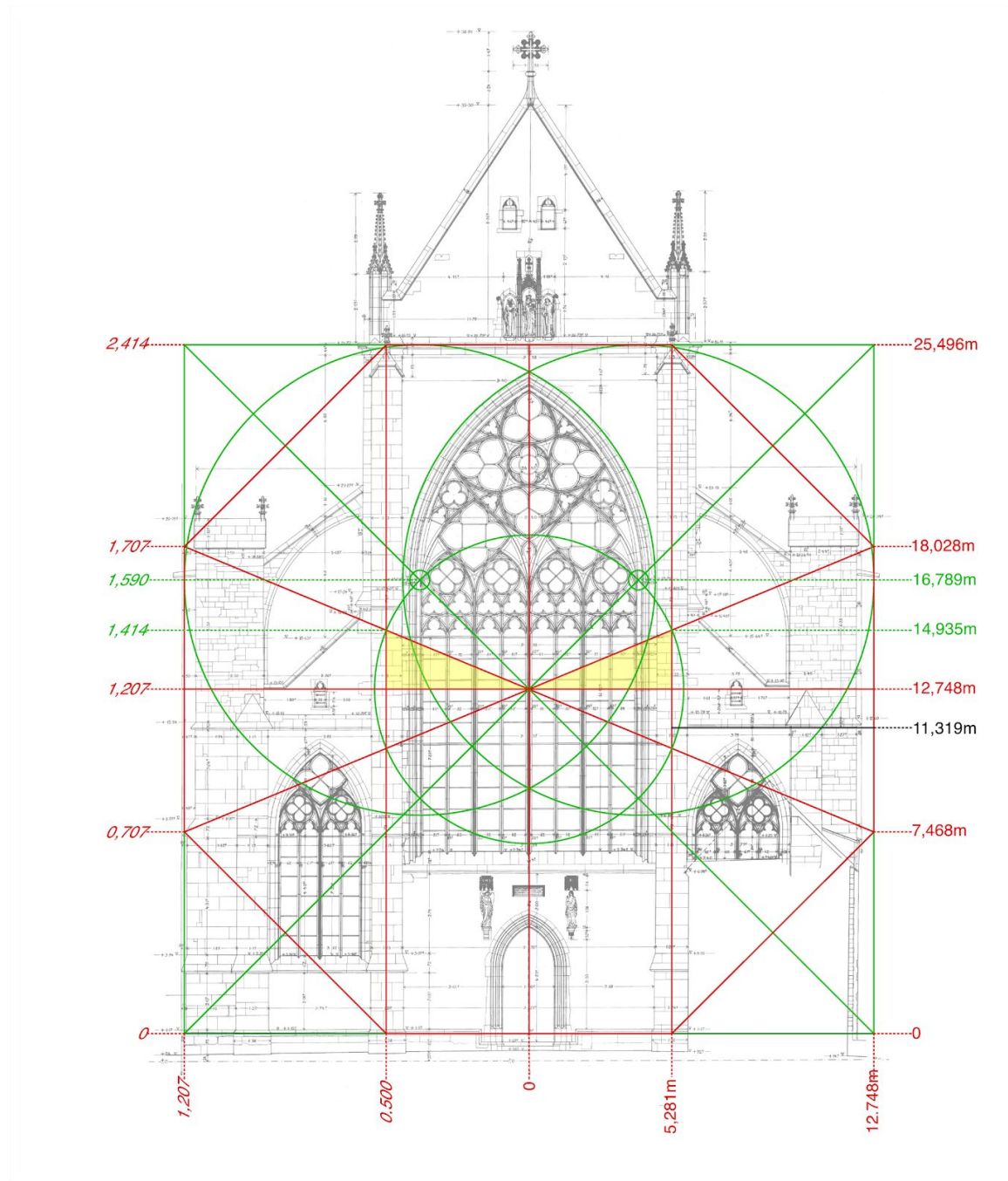


Figure 8: Altenberg, Cistercian Church, west façade with geometrical overlay, stage 1.

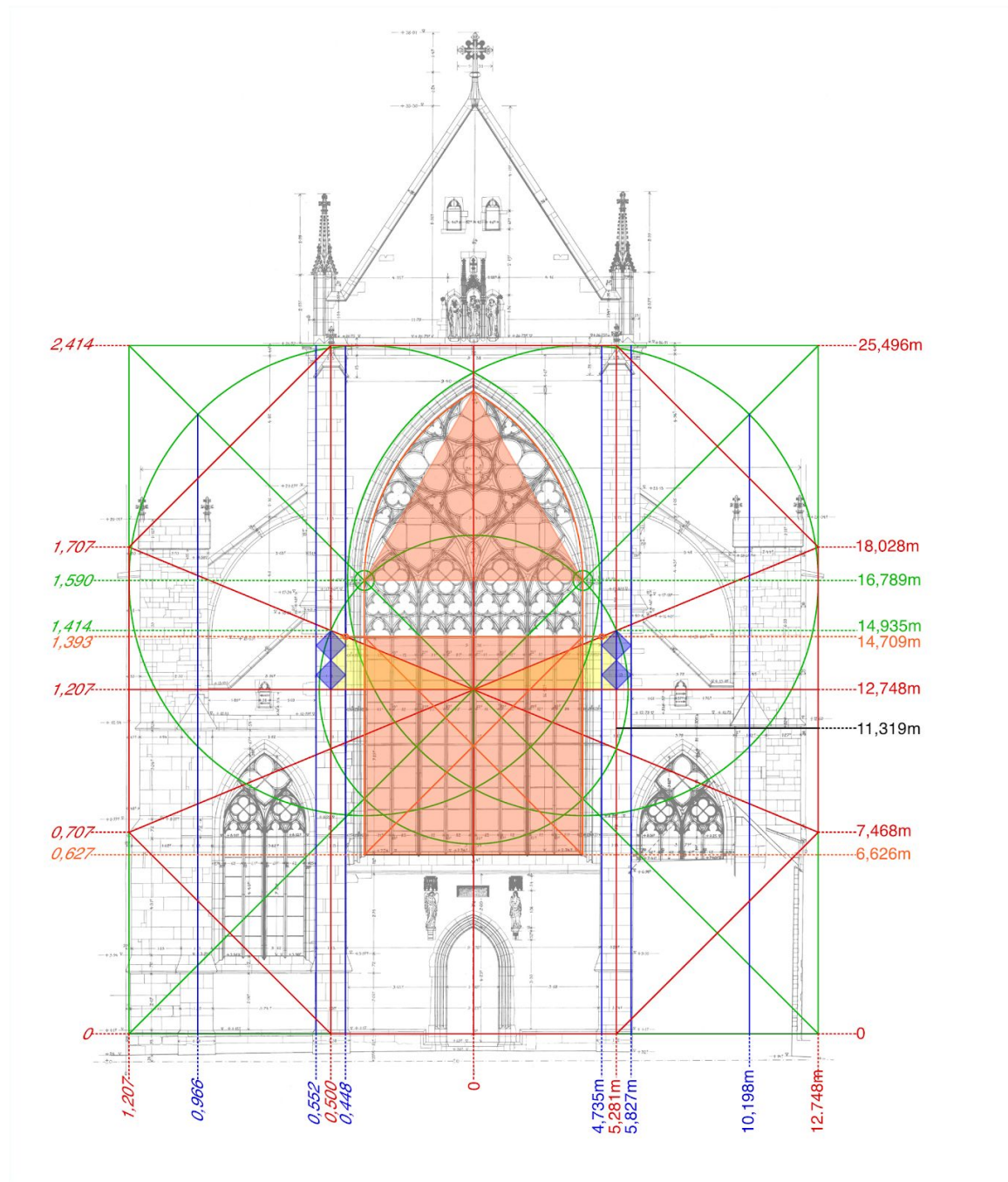


Figure 9: Altenberg, Cistercian Church, west façade with geometrical overlay, stage 2.