L'arcade actuellement visible presente l'aspect que donne le croquis que je joins à cette lettre. Le grille que j'y ai figurée ainsi que la porte en bois restent à faire.

Les parois laterals du porche comportent un enduit[1] qui va être exploré avec soin pour le cas oú il cacherait des vestiges de peinture décorative analogues à celles laissées visibles dans toute l'epaisseur du mur supprimé.

J'aurai l'honneur, Monsieur le Ministre, de vous tenir au courant de ce qui pourrait être trouvé l'intéressant au point de vue de l'art.

Je suis avec respect
Votre dévoué serviteur
D. Darcy

p.s. J'ai pu faire prendre une photographic donnant l'état de conservation des sculptures et de la peinture de tympan—je la joins à mon envoi.

1. Cet enduit est décoré d'un appareil peint à la detrempe.

Chapter 3

Changing Geometries in the North Transept of Reims Cathedral

Robert Bork

Reims Cathedral justly enjoys a reputation as one of the most unified and harmonious of major Gothic churches, but its north transept presents a number of perplexing features that seem, at least at first glance, to mark breakdowns in this overall coherence (Plate 1). The portals of the north transept, most obviously, sit within a massive masonry block that seems to crash into the rest of the composition, covering all but the tips of the three lancets below the triforium of the central bay, as seen in Figure 3.1: A.

The eastern portal, on the left in this exterior view, has a very unusual format with parallel rather than splayed jambs, suggesting to many scholars that its tympanum was initially planned for another more capacious position.[1] The lower façade also appears strikingly asymmetrical, since the right-hand tower is more massive than the left one, with thicker buttresses framing a smaller portal. Other essays in this volume carefully explore the sculptural program of the portal zone.[2] The present essay instead uses consideration of more strictly architectural idiosyncrasies in the transept façade as a springboard to geometrical analysis of the cathedral's overall design.

This study takes as its point of departure two puzzling features seen high in the north transept façade, which are also seen on its southern pendant. First, the tower stories of the transept rise markedly higher than the rose window between them, so that the terminal cornice of the façade takes on an oddly stepped profile. Second, the main transept rose is framed by a sharply pointed arch, which

1 These problems related to the north transept and its sculpture figure prominently in the scholarly literature on the cathedral, which is usefully summarized in the most recent comprehensive monograph on the cathedral's architecture, Alain Villes, *La cathédrale Notre Dame de Reims: chronologie et campagnes de travaux: essai de bilan des recherches antérieures à 2000 et propositions nouvelles* (Joué-lès-Tours: La Simarre, 2009), 25–94. Villes himself suggests that the eastern portal sculptures of the north transept, the Judgment Portal, were originally intended as a thirteenth-century addition to the cathedral's twelfth-century west façade. See ibid., 163–84.

2 See the essays in this volume by Iliana Kasarska, Jennifer M. Feltman, and Lindsey Hansen.

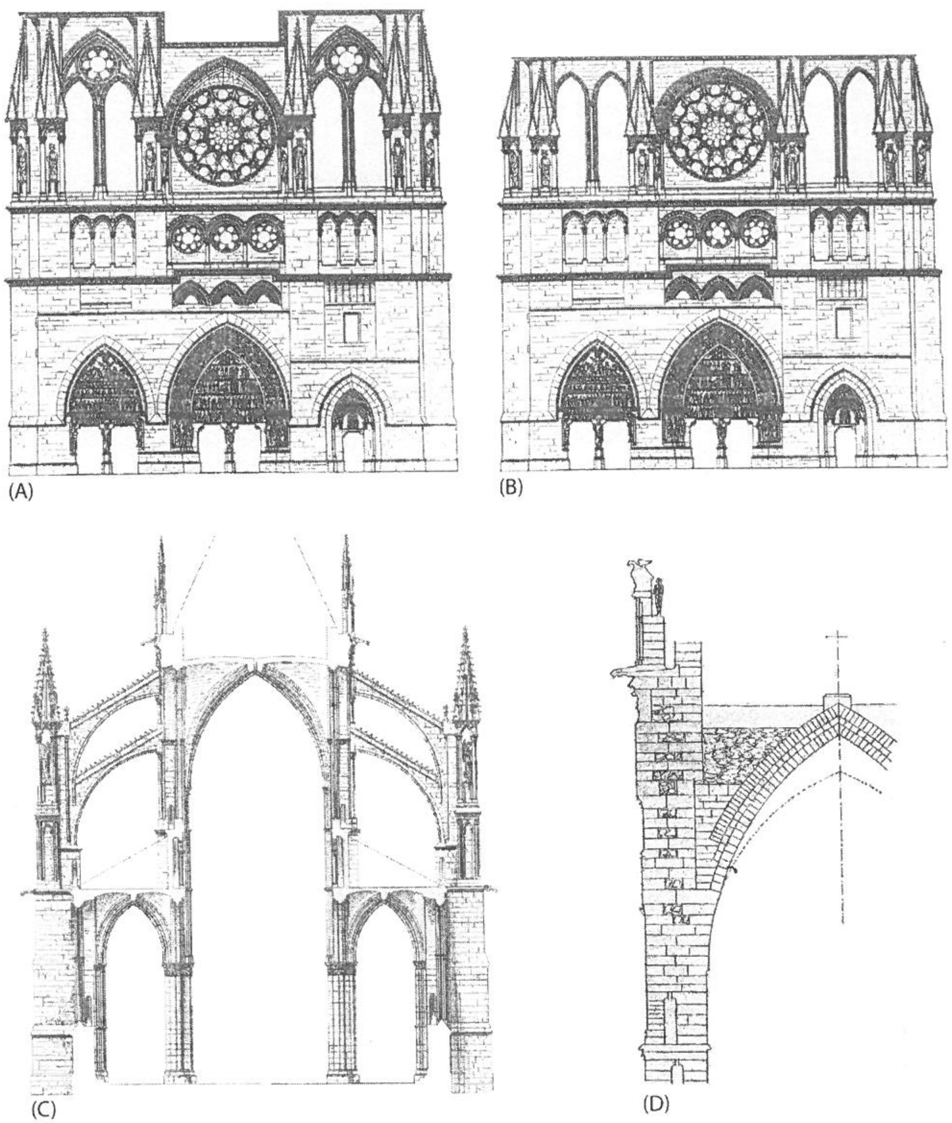

Figure 3.1 Reims, Cathedral of Notre-Dame (A) north transept façade, present elevation (source: after Thomas H. King, *The Study-book of Medieval Architecture and Art,* London: Bell and Landry, 1858, pl. 5); (B) north transept façade, hypothetical original design (source: modified by author after King, 1858); (C) nave cross section (source: after Georg Dehio and Gustav von Bezold, *Die kirchliche Baukunst des Abendlandes,* Stuttgart: J.G. Cotta, 1901, vol. 4, pl. 384); (D) original vault cross section (source: after Henri Deneux, "Des modifications apportées à la cathédrale de Reims au cours de sa construction du XIIIe au XVe siècle," *Bulletin monumental* 106 [1948]: 121–40, fig. 6)

contrasts dramatically with the round-headed arches framing the triple rosettes in the triforium story immediately below. These two anomalies together suggest that the overall format of the upper transept façade may reflect deviations from a simpler and more coherent original design. More specifically, it is tempting to imagine that the façade was originally meant to have a uniform height, with a continuous terminal molding running just slightly above a rose window framed by a round arch.[3]

Figure 3.1: B shows how such a hypothetical original design for the north transept might have looked. In this graphic, Photoshop manipulation has been used to bend the sculpted archivolts framing the rose into a regular semicircular arc. The terminal molding of the façade has been placed just slightly above this framing arch, so that it also passes just above the twin lancets of the tower bays, whose height has not been modified. Instead, they have simply been freed from the sharply pointed framing arch that now binds them together. These changes entail several other smaller ones. So, for example, the rosettes in these framing arches have to be eliminated, and the pinnacled tabernacles on the buttresses have to be shortened.[4]

The façade format proposed in Figure 3.1: B appears plausible both in itself and in comparison to other façade designs of the early thirteenth century.[5] Rose windows with round frames appear in the cathedrals of Laon, Paris, Chartres, and Amiens, and the double-lancet design proposed for the Reims tower bays closely resembles the format seen in the west façade of Amiens, the upper towers of Paris, and the transept façades of Chartres, to cite just a few of the more prominent comparanda. The double lancets in the Reims transept towers, in fact, are framed by richly crocketed mullions, which are often seen in double-lancet compositions like those just mentioned, rather than in the more unified and more explicitly window-like unified compositions involving tracery, which generally have cleaner and more streamlined mullions. The shortened tabernacles seen in Figure 3.1: B, meanwhile, actually fit their statues better than the present tall ones, and their

3 Alain Villes arrives at broadly similar conclusions in *La cathédrale Notre-Dame de Reims,* presenting various hypothetical transept designs with round-headed rose frame on pp. 379–90. His arguments, and those in the present essay, depend on the late dating for the upper transepts previously advanced by Peter Kurmann, *La façade de la Cathédrale de Reims: architecture et sculpture des portails: étude archéologique et stylistique* (Paris: Éditions du CNRS, 1987), 62–70.

4 As Villes points out in *La cathédrale Notre Dame de Reims,* 388, it is conceivable that the pinnacles were always planned to reach their present height, thus rising above the originally planned cornice. The geometrical regularity of the cathedral's overall design, however, makes it seem likely that the pinnacles were originally meant to terminate at the cornice height.

5 For a good recent survey of this design tradition, see Bernd Röder, *Türme und Fassaden von Laon bis Reims* (Trier: Roder, 2011).

shortened terminal pinnacles closely recall those of Laon Cathedral's west façade, a prototype that the leaders of the Reims Cathedral workshop surely knew.[6]

One obvious objection to the scheme shown in Figure 3.1: B is that it depicts a transept façade whose height would not match the rest of the building. More specifically, it would seem that the terminal molding of the transept façade would be too low compared to the clerestory walls of the main vessel, and that the round-headed arch over the rose would be too low compared to the high vaults, which have a sharply pointed profile, as is clearly evident in Figure 3.1: C, a cross-sectional view of the nave. This objection dissolves upon closer inspection, though, since the cathedral's high vaults were originally meant to be less pointed. The noted Reims scholar Henri Deneux observed early in the twentieth century that the springers of the cathedral's oldest high vaults have greater curvature than the later ones.[7] When this curve is continued, as in Figure 3.1: D, it becomes clear that the total height of the vault would have been approximately 1.7 m lower than in the present format. If the clerestory walls were originally meant to be correspondingly lower, as seems probable, then the overall height of the cathedral's main vessel would agree well with the hypothetical transept façade reconstruction seen in Figure 3.1: B.

In this context, the idiosyncrasies in the organization of the upper transept façades can provide valuable clues about the overall development of the cathedral's design. In addition to the obvious formal anomalies described above, there is a more subtle but very significant disjunction in the transept façades: the axes of the tabernacles flanking the rose windows do not align with the axes of the piers on the interior. More precisely, the tabernacles axes stand 13.54 m apart, while the pier axes are only 12.85 m apart.[8] The reasons for this mismatch become clear only in the context of the cathedral's overall plan.

The ground plan of Reims Cathedral has been carefully studied by Nancy Wu, who has proposed many valuable hypotheses about the geometrical logic of its conception.[9] Most crucially, she has noted that the bays adjacent to the cathedral's

6 In this shortened format, moreover, the pinnacles would start to taper at a geometrical baseline passing through the center of the rose window.

7 Henri Deneux, "Des modifcations apportées à la cathédrale de Reims au cours de sa construction du XIIIe au XVe siècle," *Bulletin Monumental* 106 (1948): 121–40, esp. 126–8.

8 These dimensions, like all the dimensions cited in the main body of this essay and its illustrations, are idealizations, based on a geometrical model built by the author using the Vectorworks computer design system. In this model, all of the main components of the building are established by the progressive unfolding of regular polygons, so that all of the dimensions are locked to each other in a single unbroken chain. The real fabric of the building, of course, lacks this absolute Platonic perfection, but the matches between the model and the measurements of the building are quite close, as comparative data in subsequent footnotes will demonstrate.

9 Wu's foundational study of the plan geometry at Reims is her 1996 Columbia University dissertation, "Uncovering the Hidden Codes: The Geometry of the East End

crossing together describe a large square, measuring some 31.02 m on a side, and she has demonstrated that simple geometrical constructions involving the diagonals and half-diagonals of this square begin to establish important dimensions in the cathedral's crossing zone and choir bays.[10] She has also underscored the importance of decagonal geometries in the cathedral's east end, where five radiating chapels gather around the five notionally equal facets of the hemicycle. In Wu's analysis, however, the links between these zones are not fully clarified, and the exterior of the building is neglected in favor of the interior.[11] The geometrical analysis of the

of Reims Cathedral." The basics of her analysis and the dimensions cited in the present study can be found in her more accessible article, "The Hand of the Mind: The Ground Plan of Reims as a Case Study" in *Ad Quadratum: The Practical Application of Geometry in Medieval Architecture,* ed. Nancy Wu (Aldershot: Ashgate, 2002), 149–68. A more recent and slightly more detailed overview of her results also appears as "Le Chevet de la cathédrale de Reims et le plan du début du XIIIe siècle," in *Nouveaux regards sur la cathédrale de Reims: actes du colloque international des 1er et 2 octobre 2004,* ed. Bruno Decrock and Patrick Demouy (Reims: Éditions Dominique Gueniot, 2008), 67–79.

10 For the subdivision of the great square to give the east–west length of the crossing bay, see Figure 6.10, in Wu, "Hand of the Mind," 167. Wu gives the precise dimensions of this "square" as 30.897 m on the east, 30.594 m on the west, 31.150 m on the south, and 31.169 m on the north. See Wu, "The Hand of the Mind," 152, n. 8. These values average to 30.953 m, while the tolerance between quantities that "should" match ranges from 2 cm in the north–south dimension to 30 cm in the east–west dimension. On this basis, a model that explains the forms of the building within tolerances of about 10 cm should be considered compelling. Wu also gives the radius to the centers of the engaged ambulatory piers as 15.51 m, in "Hand of the Mind," 151, n. 6. Since the westernmost of these engaged piers should align with the north and south faces of the great crossing square, the diameter of this circle, 2 x 15.51 m = 31.02 m, should be compared with the side lengths of the square, which average to 30.953 m. Since these values differ by less than 7 cm, they may be taken as effectively identical. Finally, since the logic of the design appears to unfold from the east, as explained below, it makes sense to treat the 31.02 m diameter of the eastern circle as geometrically fundamental.

11 In Wu's model the geometrical linkage between the straight bays and the chevet becomes tenuous. As Wu notes, and as previous scholars have observed, the geometrical center of the hemicycle lies roughly half a meter to the east of the chevet baseline. It is only by using this displaced center, whose location her model does not generate, that she can establish the length of the first straight bay in the choir. Wu herself notes, however, that the geometrical center of the engaged ambulatory piers lies closer to the baseline, and she quite reasonably suggests that only one geometrical center was originally intended. See Wu, "Le Chevet," 69–72. The model described in the present essay allows all of the bay lengths to be established using only a single geometrical center for the chevet, one that lies as it theoretically should on the east face of the straight bays. From this perspective, the distorted layout of the hemicycle can be dismissed as a slight error introduced in construction. This approach makes sense on archaeological as well as conceptual grounds because it seems that the hemicycle piers were inserted only after the exterior walls and engaged ambulatory piers had been constructed. See Walter Berry, "The Thirteenth-Century Foundations of

ground plan presented in the following paragraphs addresses these issues, while building on her pioneering work.

It makes sense to begin analysis of Reims Cathedral's ground plan at the east end, since the decagonal symmetry of the chevet establishes geometrical givens that go on to influence the proportions of other building components further to the west.[12] As Figure 3.2: A shows, layout of the chevet can start by the construction of a half-circle with a diameter of 31.02 m, matching the side length of the great crossing square identified by Wu.

A half-decagon can then be inscribed in this half-circle, and its sides can be extended to form a star whose opposite tips lie 50.20 m apart. Along the way, these sides intersect each other in points that can be taken as midpoints in the facets of another half-decagon. As the label in Figure 3.2: B indicates, each of these facets will be 11.85 m long. With this dimension in hand, it becomes possible to establish the width of the main vessel, in two simple steps. First, lines can be projected westward from the endpoints of the central one of these facets, until they intersect the rays that separate the first two pairs of chapels. Second, a semicircle concentric with the chevet as a whole can be drawn through these intersection points. This semicircle, which describes the hemicycle, has a diameter of 14.65 m.

Notre-Dame de Reims: New Evidence for the Construction History of the Cathedral," in *Arts of the Medieval Cathedrals,* ed. Kathleen Nolan and Dany Sandron (Farnham, Surrey: Ashgate, 2015), 7–25.

12 In particular, the geometry of the east end can be used to establish the width of the nave and choir vessels, a fundamental dimension whose relationship to the overall cathedral design cannot readily be explained in terms of the crossing geometry. As Wu notes, this width varies between 14.60 m and 14.88 m, slightly but distinctly smaller than the 15.51 m that one would get from simply halving the side of the great crossing square. See Wu, "Le Chevet," 69. As subsequent discussion will show, unfolding the geometry from the east gives a gratifyingly accurate width of 14.65 m. The claim that the logic of the design unfolds from the east does not imply, however, that construction had to move smoothly from east to west. The geometrical coherence of the cathedral's overall design, as revealed in the present essay, strongly suggests that the entire building plan was established by means of scale drawing before ground was broken for any actual construction. This means, therefore, that construction could in principle begin anywhere, or even at multiple sites simultaneously. So, for example, one can imagine work being undertaken simultaneously on the eastern nave bays and the radiating chapels, with the physically intermediate transept construction following only later. For variations on this scenario, see among others Kurmann, *La façade de la cathédrale de Reims,* 66, and Villes, *La cathédrale Notre-Dame de Reims,* 225–40, 391–4. To fully explore the relationship between geometry and building chronology would require going far beyond the scope of the present short essay. It is worth mentioning, however, that the coherence of the geometrical armature described here argues strongly against the dramatic revision of the transept design proposed by Villes, who suggests that each arm of the transept was originally meant to comprise three narrow bays, rather than the two comparatively large ones seen today. See Villes, *La cathédrale Notre-Dame de Reims,* 253–8.

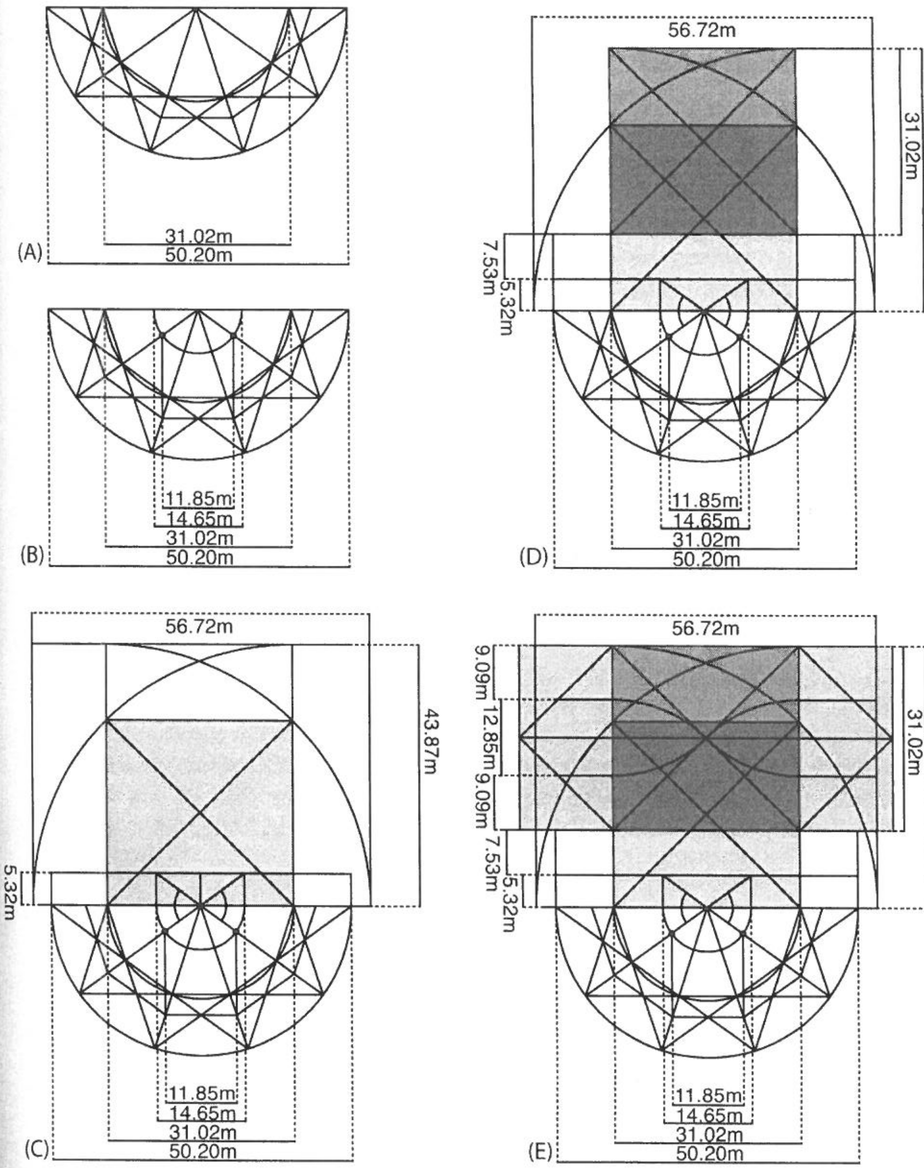

Figure 3.2 Reims, Cathedral of Notre-Dame: schematic plans showing likely steps in layout of the east end and transept zone
(drawing: Robert Bork)

This dimension thus becomes the width of the choir, and further to the west, that of the nave.[13]

With the geometry of the chevet established, the outlines of the choir and transept zone fall into place very readily, as Figure 3.2: C begins to show. The length of the easternmost straight bay, 5.32 m, can be found by simply extending the rays between the first and second pairs of radiating chapels until they intersect the axes of the choir arcades.[14] The overall footprint of the choir and transept zone, meanwhile, can be found by unfolding the diagonals of a great square 31.02 m on a side, whose edges align with the baseline of the original half-circle in the chevet. The resulting rectangle measures 56.72 m from north to south, and 43.87 m from east to west.[15] These dimensions agree superbly well with those measured in the building. In fact, the total length of 43.87 m described here matches the corresponding length in the building almost to the centimeter.[16]

13 The theoretical nave width of 14.65 m described here works almost perfectly close to the crossing bay, which measures 14.65 m across its western face and 14.69 m across its eastern face, according to Wu, "Hand of the Mind," 153, n. 11. The main vessel swells slightly toward the east, so the first piers of the hemicycle stand just over 14.80 m apart, measured from center to center. Wu, "Hand of the Mind," 154, gives this dimension as 14.88 m, but on page 151, n. 6, she gives the best-fit radius of the hemicycle as 7.41 m, which would give a theoretical span of 14.82 m. Hamann-MacLean and Schüssler give the hemicycle width as 14.80 m, and the crossing bay widths as 14.69 m and 14.68 m on east and west, respectively; see Richard Hamann-MacLean and Ise Schüssler, *Die Kathedrale von Reims* (Stuttgart: Franz Steiner Verlag, 1993); their diagram with these bay lengths is reproduced in Villes, *La cathédrale Notre-Dame de Reims,* 253. If indeed the 14.65 m dimension was intended based on scaling up from a drawing, as the present analysis suggests, this would imply that the crossing bay was built more precisely than the hemicycle.

14 In mathematical terms, $5.32\text{ m} = 0.5 \times 14.65\text{ m} \times \tan(36°)$, where $36° = 180° / 5$ is the width of each wedge in the chevet composition.

15 In mathematical terms, $43.87\text{ m} = 31.02\text{ m} \times \sec(45°) = 31.02\text{ m} \times \sqrt{2}$. Similarly, $56.72\text{ m} = 31.02\text{ m} \times (2\sqrt{2} - 1)$.

16 According to Hamann-MacLean and Schüssler, the bay lengths on the south side of the arcade, starting from the east, are 5.33 m, 7.40 m, 9.24 m, 12.92 m, and 9.00 m, for a total of 43.89 m. This is virtually identical to the dimensions in the model proposed in this essay, which are 5.32 m, 7.53 m, 9.09 m, 12.85 m, and 9.09 m, for a total of 43.87 m. The south side is best for this dimensional comparison, because the easternmost pier on the north side is displaced some 30 m eastward into the hemicycle, as comparison of all the adjacent bay lengths demonstrates. See the graphic of bay lengths measured by Hamann-MacLean and Schüssler, and reproduced in Villes, *La cathédrale Notre-Dame de Reims*, 253. It is interesting and significant that the overall length of this great rectangle agrees so precisely with its theoretical value, despite the variance of bay dimensions within that frame. The two aisles flanking the transept, for example, have widths varying between 9.00 m and 9.24 m, which averages to 9.12 m, instead of the uniform width of 9.09 m that they have in the model. One may imagine, therefore, that the western aisle was "trimmed" down to just 9.00 m, to compensate for the overly large eastern aisle, so that the composition as a whole would fit neatly into the carefully controlled frame of the great rectangle.

When the great crossing square identified by Nancy Wu is inserted within this geometrical frame, as in Figure 3.2: D, the remaining lengths in the straight choir bays begin to emerge automatically. When the 31.02 m length of this square and the 5.32 m length of the easternmost straight bay are subtracted from the 43.87 m length of the whole composition, a bay measuring 7.53 m from east to west is left over. And, when the half-diagonals of the crossing square are unfolded in a scheme first proposed by Wu, the crossing square gets partitioned into three bays measuring 9.09 m, 12.85 m, and 9.09 m from east to west, as Figure 3.2: E shows.[17] This is the origin of the previously mentioned 12.85 m transept width, measured between pier axes. To understand the mismatch between this dimension and the 13.54 m between the exterior pinnacle axes in the upper transept, consideration will have to be given to the cathedral's overall elevation.

Before going on to explore the vertical plane, it is worth noting how effectively the simple crystalline geometry shown in Figure 3.2: E describes the actual plan of the building, which appears in Figure 3.3.

As the latter image shows, the 50.20 m span of the circle developed in Figure 3.2: A sets the width to the outer wall surfaces of the straight bays. The 56.72 m span of the great rectangle established in Figure 3.2: C sets the width to the outer buttress surfaces in the straight bays. And a simple doubling of the great 31.02 m crossing square gives the dimension across the exterior surfaces of the transept buttresses, except for those in the east tower of the north transept, which are slightly smaller than all the others.[18] Other, more detailed, geometrical systems can be developed within this framework to describe features such as radiating chapels and the buttresses between them, but the basic relationships already described suffice to determine most features of the ground plan in the transept zone.[19]

The design of the transept façades at Reims Cathedral depends in an interesting way on the design of the building's main vessel. In churches that have square crossings, the cross-vessel defined by the transept is geometrically equivalent to

17 Wu, "Hand of the Mind," 152, 167.

18 These relationships all work satisfyingly well when judged against the ground plan published by Deneux in 1944, as Figure 3.3 shows. The building's exterior outlines are carefully recorded in this graphic, but they admittedly appear with less precision than they would in a modern laser-based survey. At this writing, a comprehensive laser survey of the cathedral remains to be undertaken. The nineteenth-century exterior elevations that serve as the bases for Figures 3.4 and 3.5, therefore, also have yet to be decisively superseded. Fortunately, these graphics are precise enough to develop geometrical hypotheses that can be tested against increasingly precise measurements of the cathedral as they become available.

19 The radiating chapels at Reims have a great deal in common with those of Suger's St-Denis, whose geometry is discussed in Robert Bork, "Ground Plan Geometries in Suger's St-Denis: A Prototype for Altenberg," in *Aufmass und Diskurs,* ed. Astrid Lang and Julian Jachmann (Berlin: Lukas Verlag, 2013), 55–68.

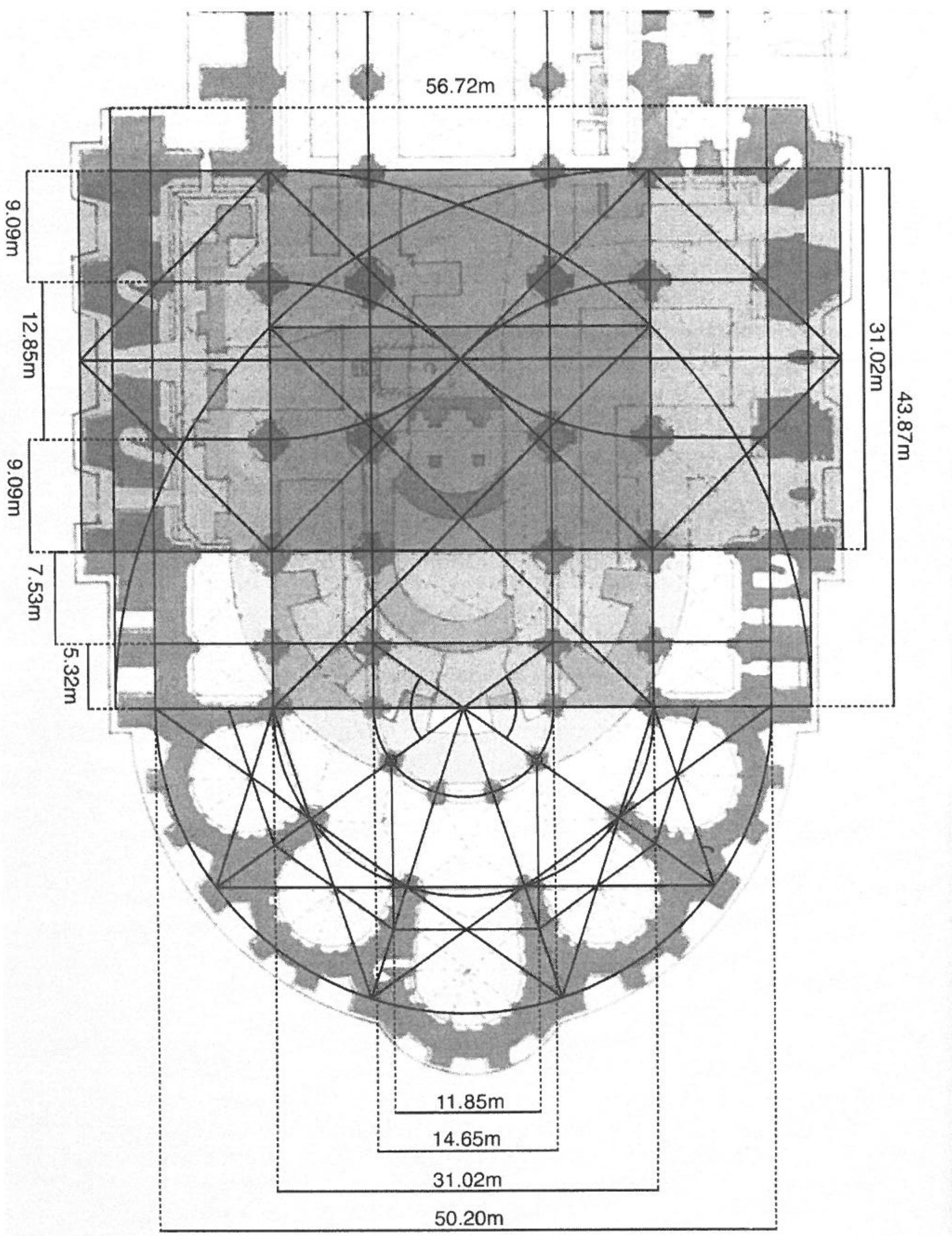

Figure 3.3 Reims, Cathedral of Notre-Dame: plan geometry of the cathedral's east end and transept zone

(source: schematics by the author, overlaid on survey plan by Henri Deneux, *Dix ans de fouilles dans la cathédrale de Reims,* Reims, 1944, frontispiece)

the longitudinal vessel comprised by the nave and choir. At Reims, though, the crossing bay is rectangular, so that the transept measures only 12.85 m across, while the nave measures 14.65 m across, measuring in both cases between pier axes.[20] Clearly, therefore, a proportional analysis based on the transept width will unfold rather differently than one based on the nave width. Since the nave and choir have greater liturgical and architectural primacy than the transept, there is good reason to imagine that the longitudinal vessel has greater geometrical primacy, as well. As subsequent discussion will reveal, in fact, the cross section of the Reims nave was set by a simple octagon-based geometrical framework, which established heights that would go to affect the design of the transept façades.

Figure 3.4 shows a cross section of the Reims Cathedral nave, and the geometrical armature that appears to have governed its design.[21]

The fundamental dimension in this diagram is the 14.65 m span between the axes of the nave arcade. A great octagon with facets of this length rises from the nave floor to height 35.36 m. This height at first seems to match nothing of significance in the section, but it corresponds quite precisely to the height of the originally planned vaults, as described by Deneux. The center of the octagon, moreover, falls at height 17.68 m, perfectly aligned with the top of the arcade zone and the baseline of the triforium. This is already an interesting result, since it suggests that the cathedral's arcade was meant to have exactly half the height of the originally planned main vessel.[22] But it is possible to go much further with the

20 The actual dimensions of the crossing bay, according to Wu, are 14.69 m east face, 14.65 m west face, 12.92 m north face, and 12.90 m south face. See Wu, "Hand of the Mind," 153 n. 11. Hamann-MacLean and Schüssler give 14.69 m east face, 14.68 m west face, 12.93 m north face, and 12.92 m south face. See their diagram reproduced in Villes, *La cathédrale Notre-Dame de Reims,* 253.

21 The octagon-based cross section of the Reims nave would prove to be highly influential. Very similar proportioning schemes would be seen in the thirteenth century in the so-called Reims Palimpsest, at the cathedral of Clermont-Ferrand, at the Liebfrauenkirche in Trier, and at the Cistercian church of Altenberg; they remain popular in the fourteenth century, as seen in the cathedral choirs of Prague and Aachen. Many other examples could surely be added to this list, which includes only examples so far studied by the author. On the Reims Palimpsest, Prague, and Clermont-Ferrand, see Robert Bork, *The Geometry of Creation*, 42–53, 201–14, and 390–400, respectively. On Trier, see Robert Bork, "Geometrie, Proportionen, und Vermessungen in der Liebfrauenkirche," in *Liebfrauen in Trier – Architektur und Ausstattung von der Gotik bis zur Gegenwart,* ed. Stefan Heinz and Andreas Tacke (Petersberg: Imhof, 2016), 187–98. On Altenberg, see Robert Bork, Sabine Lepsky, and Norbert Nussbaum, *Gotische Konstruktion und Baupraxis an der Zisterzienserkirche Altenberg, Band 2 Quer- und Langhaus* (Bergisch Gladbach: Altenberg Dom-Verein, 2012), 75–88. On Aachen, see Robert Bork and Norbert Nussbaum, "Gotischer Baubetrieb am Aachener Münsterchor," in *Sie glänzte wie ein kostbarer Edelstein, wie ein kristallklarer Jaspis. 600 Jahre Aachener Chorhalle, Karlsverein–Dombauverein 16* (Aachen: Dombauverein, 2014), 22–42.

22 It is interesting in this connection that Villard de Honnecourt's drawings of Reims Cathedral show the arcades reaching half the total height of the main vessel, since this suggests

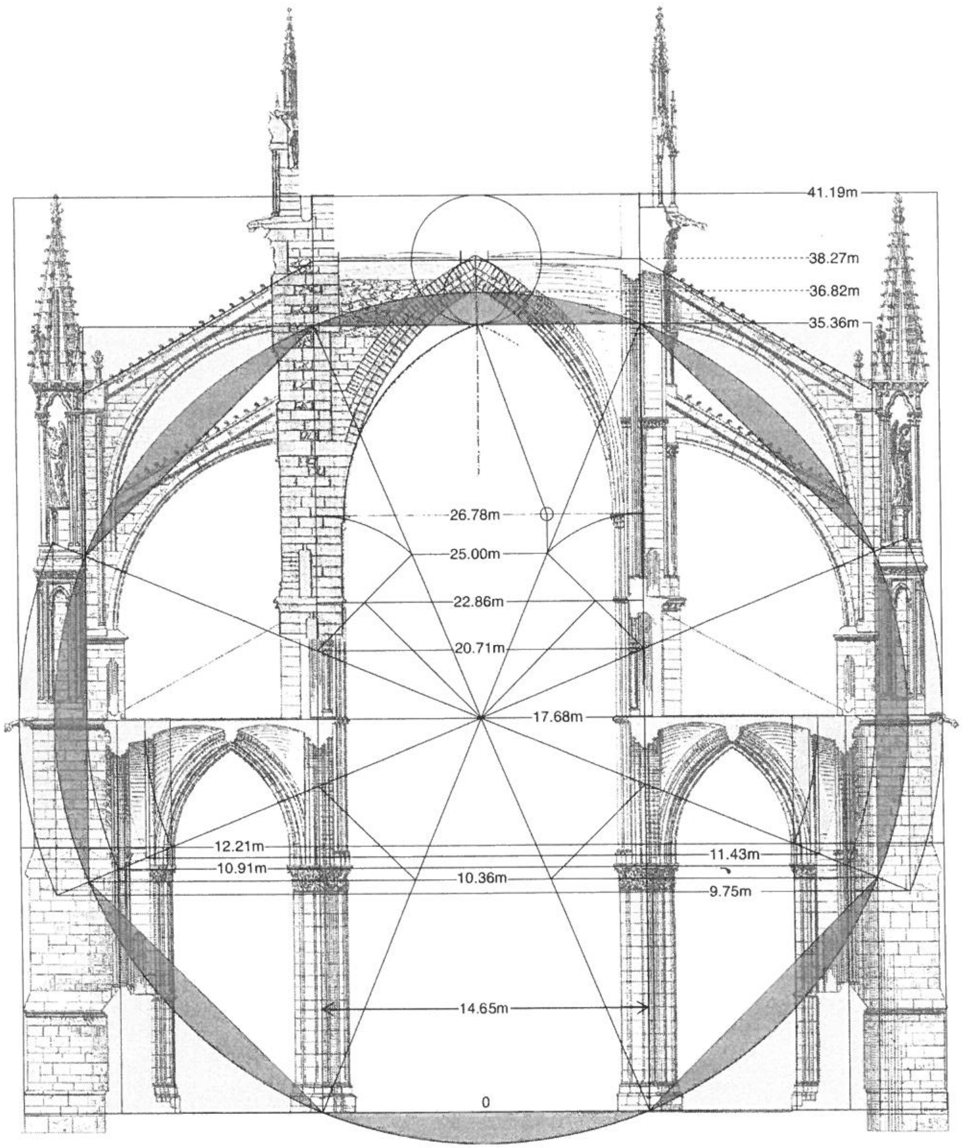

Figure 3.4 Reims, Cathedral of Notre-Dame: geometry of the nave cross section (source: schematics by the author, overlaid on section from Dehio and Bezold, *Die kirchliche Baukunst des Abendlandes,* 1901)

geometry of the great octagon and its derivatives. The lower lateral corners of the octagon, for example, fall at height 10.36 m, the level at which the capitals spring from the wall piers in the aisles.

To go further, it is necessary to inscribe further octagons concentric with this original one. So, for example, a small octagon of this type framed by the arcade axes fills the space between heights 10.36 m and 25.00 m. Its upper lateral corners fall at height 20.71 m, the level at which the small columns in the triforium terminate. The midpoints of the small octagon's upper diagonal facets fall at height 22.86 m, which marks the top edge of the triforium. When these diagonal facets are unfolded, they reach up to height 26.78 m, the level from which the main vaults spring from the tops of their capitals. The geometrical center of the originally planned vault curvature, moreover, is a point at this level located directly above the corner of the small octagon; this point is picked out by a small open circle in Figure 3.4. The radius of curvature in the originally planned vault corresponds to the distance between this center point and the midpoint of the great octagon's upper facet, as the sloping line between these two points in the graphic indicates.

The section of the Reims nave, like many other Gothic designs, incorporates geometrical relationships based on the inscribing of octagons within each other. It is convenient to call this practice octature, by analogy to the better-known practice of quadrature, or square rotation, which is familiar to many students of Gothic architecture from medieval documents such as the thirteenth-century "portfolio" of Villard de Honnecourt and the fifteenth-century pinnacle design booklet of Mathes Roriczer.[23] In their illustrations of quadrature, the midpoints of a starting square are used as the corner points of a rotated square inscribed within it, as in Figure 3.5: A.

A precisely equivalent sequence can be produced by inscribing a circle within the starting square, and a square within the circle, as in Figure 3.5: B. As this graphic shows, the length of the inscribed square will be 0.707 as great as that of the framing square. In modern mathematical parlance, this factor of 0.707 can be described as the cosine of the 45-degree angle between the square's equator and its corner, but medieval draftsmen needed no such vocabulary to make these constructions with their compasses and straightedges. They were just as capable of making the analogous construction based on the octagon. As Figure 3.5: C shows, the diameter of the inscribed octagon will be 0.924 as great as that of the framing

that he was recording the original design. Villard's drawings in general appear far less precise than the later medieval "blueprints" that survive mainly in the German-speaking world, but they actually have more geometrical coherence than many scholars have supposed. On this aspect of Villard's production, see Bork, *The Geometry of Creation,* 31–42 and Villes, *La cathédrale Notre-Dame de Reims,* 507–36. On Villard more generally, see Carl F. Barnes, *The Portfolio of Villard de Honnecourt* (Farnham, Surrey: Ashgate, 2009).

23 See Lon Shelby, *Gothic Design Techniques: The Fifteenth-Century Design Booklets of Mathes Roriczer and Hans Schmuttermayer* (Carbondale: Southern Illinois Press, 1977).

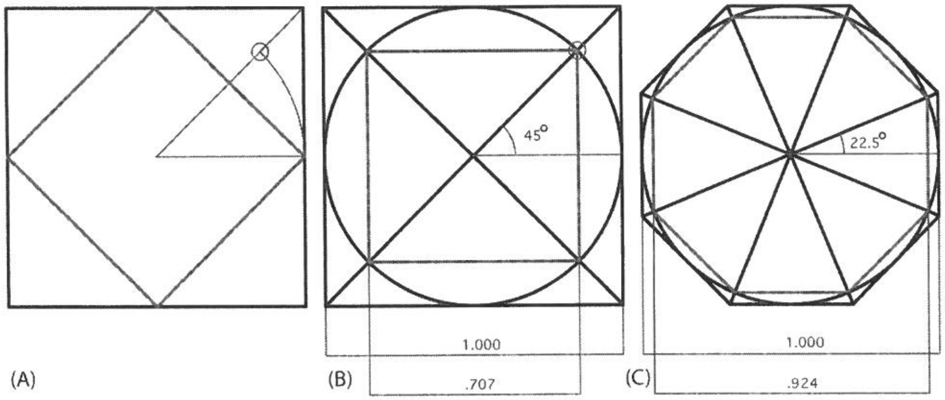

Figure 3.5 Square rotation, quadrature, and octature
(drawing: Robert Bork)

octagon, where this factor of 0.924 is the cosine of the 22.5-degree angle between the octagon's equator and its adjacent corner. By repeated use of this octature scheme, it is possible to construct sequences of progressively smaller octagons nested within each other, just as one can do with squares using quadrature.[24]

The shading in Figure 3.4 shows how this system of nested octagons was used to help establish the cross section of the Reims Cathedral nave. The darkest shading shows the circle circumscribed around the original great octagon, whose lower facet corresponds to the floor of the main nave vessel. As noted previously, the lower lateral corners of this great octagon fall at height 10.36 m, aligning with the bottom edges of the capitals on the wall piers in the aisles. When a larger octagon is circumscribed about this one by octature, its lower lateral corners fall at height 9.75 m, aligning with the bottom edges of the main arcade piers. As the lightly shaded arcs on the outer margins of Figure 3.4 show, the next octature step outwards from the dark circle gives the full width of the cathedral's cross section, measured across its outer buttress surfaces at ground level. Moving inwards, the first set of lightly shaded arcs within the original dark one shows that the next smaller octagon will have corners at height 10.91 m, aligning with the top of the aisle capitals; in the vertical dimension, meanwhile, these same corner points align precisely with the plane of the aisle windows. Moving inwards by two more octature steps, one finds the corner points at height 12.21 m, the level marking the top edge of the diagonal setbacks on the nave buttresses; these same corners align in the vertical dimension with the inner faces of the engaged wall piers. It is absolutely clear, therefore, that a system of nested octagons governed the section

24 Both octature and quadrature were already being used at Jumièges abbey in the eleventh century. See Robert Bork, "La géométrie de Notre-Dame de Jumièges," in *Notre-Dame de Jumièges,* ed. James Morganstern (forthcoming).

of the Reims nave in both the horizontal and vertical dimensions, at least in the zone up to the base of the triforium.

Extensions of this octagon-based system appear to have governed the design of the upper nave zone as well, although the modification of the vault heights seems to have introduced some slight inconsistencies into the system. So, for example, the present vaults rise almost exactly to height 36.82 m of the darkly shaded circle around the original great octagon. In fact, they rise a tiny bit higher, because their curvature was set by a simple construction based on the equilateral triangle; the distance from the springing of the vault to its vertex equals the free span between the springers. The interval between the great octagon and its circumscribing circle nevertheless seems to have played a role in the layout of the upper nave, since a doubling of this interval above the octagon reaches height 38.27 m, where a molding on the outer wall marks the top of the main vessel proper, while a quadrupling of the interval reaches height 41.19 m, a level that coincides with the top level of the stone wall structure and with the tips of the pinnacles on the outer pier buttresses. This height also corresponds quite closely with the width of the structure measured at ground level, suggesting that the whole composition was meant to fit neatly within a square.[25]

The preceding analyses of the cathedral's ground plan and nave section provide the keys to understanding the geometry of transept façades, including the curious axis shifts noted near the beginning of this essay. Figure 3.6 reveals that the overall width of the upper transept is 35.36 m, which is the diameter of the great octagon first described in the nave.

25 The width of the buttresses seems to have been set as 41.43 m, which is two octature steps larger than the 35.36 m size of the original generating square: 41.43 m = 35.36 m / $(0.924)^2$. Further evidence for the importance of the 35.36 m square comes from the shape of the fifteenth-century roof, which can still be read in the west façade gable, despite the destruction of the roof itself in WWI. The height of the roof seems to have been roughly 57 m, which is to say that it was greater than the side length of the 35.36 m square by a factor very closely approximating the so-called Golden Section, ϕ, which can be found geometrically by unfolding the half-diagonal of a square. In mathematical terms, ϕ satisfies the equation $\phi = 1 / (\phi - 1)$, and it has the value $(1 + \sqrt{5}) / 2 = 1.618\ldots$ Its importance for Gothic design has been effectively demonstrated by authors including Stephen Murray, who sees it as a crucial generator for the plan geometry of Amiens Cathedral, and Peter Kidson, who documents its use at Salisbury. See Murray and James Addiss, "Plan and Space at Amiens Cathedral: With a New Plan Drawn by James Addiss," *Journal of the Society of Architectural Historians* 49 (1990): 44–66; and Peter Kidson, "The Historical Circumstance and the Principles of the Design," in *Salisbury Cathedral: Perspectives on the Architectural History,* ed. Thomas Cocke and Peter Kidson (London: HMSO, 1993), 31–97, esp. 62–75. In this context, it is tempting to imagine that the original roof at Reims may have been intended to reach the geometrically elegant height of 57 m, although Villes argues that the original thirteenth-century roof was probably lower. See Villes, *La cathédrale Notre-Dame de Reims,* 615–20.

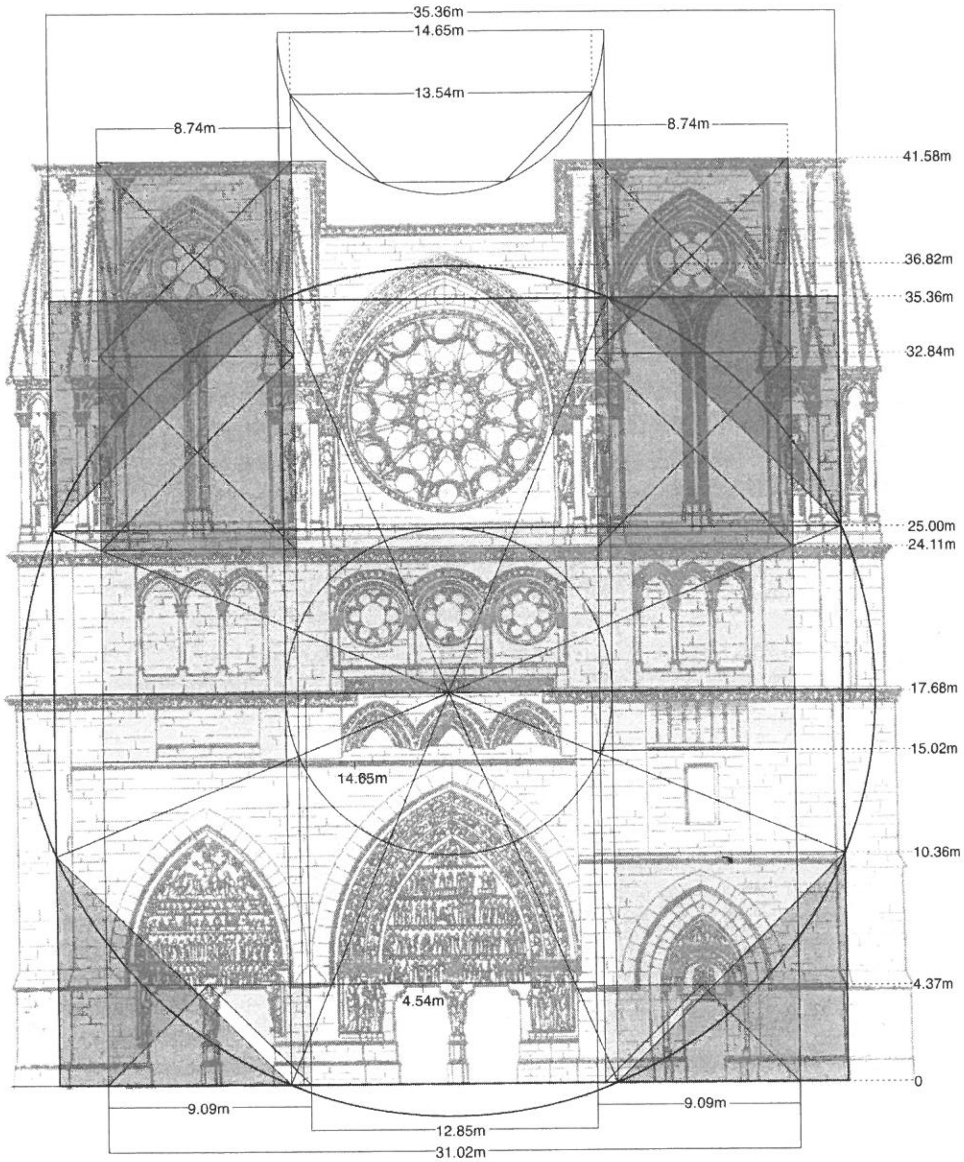

Figure 3.6 Reims, Cathedral of Notre-Dame: geometry of the north transept elevation
(source: schematics by the author, overlaid on elevation from King, 1858)

The facets of this octagon, as noted previously, measure 14.65 m, matching the distance between the axes of the nave piers. As the top of the figure shows, an octagon inscribed within a circle of this diameter will have a diameter of 13.54 m, which is the distance between the pinnacle axes in the upper transept. There are, therefore, two distinct vessel widths in the transept: the 12.85 m width between the inner pier axes measured at ground level, which was determined by unfolding the half-diagonals of the great 31.02 m crossing square in the plan; and the 13.54 m width between the upper pinnacle axes, which relates via octature to the 14.65 m interaxial width of the nave. In a similar vein, the graphic reveals the origins of two other main widths in the transept façade: the 31.02 m width between the outer pier axes at ground level, and the 35.36 m width across the body of the upper façade, which is the diameter of an octagon whose facets match the 14.65 m nave width. This analysis shows, therefore, that the axis shifts noted previously were by no means haphazard; rather, they resulted from the interlocking of the carefully established crystalline geometries of the building plan and the nave elevation.

While the horizontal dimensions in the north transept façade attest to the substantially unified planning of the cathedral as a whole, consideration of its vertical dimensions reveals a number of interesting idiosyncrasies related to changes in the design, of which the adjustment in vault height was the most important. Comparably consequential, especially in the lower reaches of the façade, was the singular massiveness of the western tower, which shelters a portal far smaller than the central and eastern ones. In the central and eastern portals, the tympana begin at height 4.54 m, the same level marked by the horizontal molding on the eastern buttress. This height is simply half of the 9.09 m width of the eastern transept aisle measured between pier axes at ground level, which was seen already in Figures 3.2 and 3.3. On the western tower, by contrast, the corresponding horizontal molding runs at height 4.37 m, which is precisely half the 8.74 m width of the western tower, measured this time between its upper pinnacle axes. So, while the eastern tower could have been begun before the shifted pinnacle axes of the upper transept were decided upon, even the lower stories of the western tower could have been begun only after this decision. This geometrical evidence thus provides independent confirmation of the west tower's late relative dating, which previous scholars have already proposed on archaeological grounds.[26]

In the zone around the middle of the north transept façade, the arrangement of horizontal moldings attests to the importance of the octagon-based geometrical armature already described in the nave section. Its importance is most immediately obvious in the west tower, where a prominent molding runs at height 10.36 m, aligning with the corner of the great octagon, and thus with the bottom edges of the capitals on the wall piers of the interior. The next horizontal in the west

26 See, for example, Villes, *La cathédrale Notre-Dame de Reims,* 151–61. On the subsurface archaeology of the transept zone and its implications for the relative dating of its components, see the essay by Walter Berry in this volume.

tower falls at height 15.02 m, where one of the rays from the octagon's center to that corner intersects the vertical axis of the interior piers. The corresponding horizontal above the eastern and central portals falls at height 14.65 m, where the analogous ray cuts the vertical axis of the exterior pinnacles; this dimension also equals the interaxial width of the nave vessel.

The equator of the octagon falls at height 17.68 m, corresponding to the floor of the triforium on the interior. On the exterior, the molding across the middle of the north transept façade jumps subtly from just below this level in the towers to just above it in the central bay, probably to provide a little bit of extra room for the three lancet windows whose tips just manage to peek out above the masonry block framing the central portal. It is noteworthy that the lateral buttress on the east face of the façade aligns perfectly with the circle circumscribing the great octagon. On the western side of the façade, though, this relationship no longer pertains so neatly, because all of the tower buttresses at this level are still thicker than those on the east. For the same reason, the arches in the west tower triforium are compressed, being slightly narrower than those in the east tower. The foliate molding crowning the triforium terminates at height 24.11 m, where the rays to the octagon corners cross the axes of the outer pinnacles, which are 31.02 m apart, corresponding to the size of the great squares seen in the ground plan.

The organization of the transept façade above the triforium seemingly bears witness to changes of program associated with the changing of the vault profile. The current sharply pointed frame around the rose window contrasts oddly with the round-headed arches over the rosettes in the triforium. As noted previously, therefore, it is tempting to imagine that the originally contemplated transept design would have featured a round frame around the rose, as seen at Laon, Paris, Chartres, and Amiens. As Figure 3.6 shows, after all, the rose window fits neatly within the geometrical armature defined by the great octagon. The top of the octagon, at height 35.36 m, also corresponds to two other significant features in the upper transept façade: the tips of the twin lancets in the towers, and the bases of the slender colonnettes on the corners of the upper buttresses. On both aesthetic and geometrical grounds, therefore, it seems likely that the façade was originally meant to end just above this level, as illustrated in Figure 3.1: B.

In practice, of course, the towers of the north transept were built higher than the rose zone in between, creating the oddly stepped profile seen today.[27] Here again, a geometrical perspective makes clear the rationale for this format. The twin arches in the tower lancets spring at height 32.84 m, which—allowing for rounding—is exactly 8.74 m above the 24.11 m level at the top of the triforium. Since 8.74 m is the width of the upper tower measured between pinnacle axes, this means that the tower between the triforium and the lancet heads is a perfect square. Stacking another such square on top of the first, one reaches height 41.58 m, where

27 On this zone, see Villes, *La cathédrale Notre-Dame de Reims,* 379–90; and Kurmann, *La façade de la cathédrale de Reims,* 62–70.

the tower now terminates. The upper towers in their present format, therefore, are perfect double squares in elevation. They are tall enough to accommodate the large rosettes and sharply pointed framing arches that now surmount the twin lancets. Since there would have been no need for such tall tower stories prior to the transformation of the vault curvature, it seems likely that the original design originally called for the lancets to stand by themselves in a wall ending at the same height as the wall over the rose window, creating the simpler design shown in Figure 3.1: B.

In sum, therefore, geometrical analysis of the formal anomalies in the Reims north transept can reveal a great deal about the formal logic of the entire cathedral. The bay lengths seen in the transept interior at ground level, for example, derive directly from the unfolding diagonals of a great square around the crossing, as Nancy Wu already demonstrated. The pinnacle spacing on the upper transept exterior, meanwhile, relates via octature to the interaxial span of the cathedral's nave, which can be related to the great crossing square, in turn, through the decagonal geometry of the chevet. In elevation, too, many of the crucial heights in the transept façade relate directly to those seen in the cross section of the nave, which is governed by a great octagon and its immediate derivatives. All of these components of the design fit together with a beautiful crystalline clarity, suggesting that the main outlines of the cathedral's plan and elevation were already fully determined at the start of construction. Within this framework, though, modifications were introduced, most notably to accommodate vaults sharper and taller than those originally foreseen.[28] These changes are most evident in the north transept façade, whose unusual format thus provides valuable information about the evolving design of the cathedral as a whole.

28 The cathedral's west façade appears far more harmonious than the north transept façade because it was designed as a coherent unit, and late enough to take all of these adjustments into account. For the dating of the west façade design to the mid-thirteenth century, see esp. Kurmann, *La façade de la cathédrale de Reims,* esp. 130–59, and Villes, *La cathédrale Notre-Dame de Reims,* 417–507. The geometry of the west façade, which involves elegant extensions of the octagon-based system seen in the nave elevation, will be discussed in the forthcoming dissertation by Rebecca Avery Smith, written under the supervision of the author at the University of Iowa. See also Robert Bork, "The Linked Geometries of Reims Cathedral's Nave Section and West Façade," *AVISTA Forum Journal* 23 (2013): 78–84.